Section 1.1

In each of Problems 1 through 6 draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe the dependency.

1. $y' = 3 - 2y$.
2. $y' = 2y - 3$.
5. $y' = 1 + 2y$.

Section 1.2

1. Solve each of the following initial value problems and plot the solutions for several values of y_0. Then describe in a few words how the solutions resemble, and differ from, each other.

(a) $dy/dt = -y + 5, \quad y(0) = y_0$
(b) $dy/dt = -2y + 5, \quad y(0) = y_0$
(c) $dy/dt = -2y + 10, \quad y(0) = y_0$.

2. Follow the instructions for Problem 1 for the following initial value problems.

(a) $dy/dt = y - 5, \quad y(0) = y_0$
(b) $dy/dt = 2y - 5, \quad y(0) = y_0$
(c) $dy/dt = 2y - 10, \quad y(0) = y_0$.

7. The field mouse population in Example 1 satisfies the differential equation

$$dp/dt = 0.5p - 450.$$

(a) Find the time at which the population becomes extinct if $p(0) = 850$.
(b) Find the time of extinction if $p(0) = p_0$, where $0 < p_0 < 900$.
(c) Find the initial population p_0 if the population is to become extinct in 1 year.

9. The falling object in Example 2 satisfies the differential equation

$$dv/dt = 9.8 - (v/5), \qquad v(0) = 0.$$

(a) Find the time that must elapse for the object to reach 98% of its limiting velocity.
(b) How far does the object fall in the time found in part (a).

12. A radioactive material, such as the isotope thorium-234, disintegrates at a rate proportional to the amount currently present. If $Q(t)$ is the amount present at time t, then $dQ/dt = -rQ$, where r is the decay rate.
(a) If 100 mg of thorium-234 decays to 82.04 mg in 1 week, determine the decay rate r.
(b) Find an expression for the amount of thorium-234 present at any time t.
(c) Find the time required for the thorium-234 to decay to one-half its original amount.

13. The **half-life** of a radioactive material is the time required for an amount of this material to decay to one-half its original value. Show that for any radioactive material that decays according to the equation $Q' = -rQ$, the half-life τ and the decay rate r satisfy the equation $r\tau = \ln 2$.

Section 1.3

In each of Problems 1 through 6 determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. $t^2 d^2y/dt^2 + t dy/dt + 2y = \sin t$
3. $d^4y/dt^4 + d^3y/dt^3 + d^2y/dt^2 + dy/dt + y = 1$
4. $dy/dt + ty^2 = 0$
6. $d^3y/dt^3 + t dy/dt + (\cos^2 t)y = t^3$

In each of Problems 7-14, verify that the given functions are solutions of the differential equation.

8. $y'' + 2y' - 3y = 0, \quad y_1(t) = e^{-3t}, \quad y_2(t) = e^t$
13. $y'' + y = \sec t, \quad 0 < t < \pi/2; \quad y = (\cos t) \ln \cos t + t \sin t.$

18. Determine the values of r for which the differential equation $y''' - 3y'' + 2y' = 0$ has solutions of the form e^{rt}.

Section 2.1

In each of Problems 1-12: (a) Draw a direction field for the given differential equation. (b) Based on inspection of the direction field, describe how solutions behave for large t. (c) Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$.

2. $y' - 2y = t^2 e^{2t}$
8. $(1 + t^2)y' + 4ty = (1 + t^2)^{-2}$

13. Find the solution of the initial value problem: $y' - y = 2te^{2t}, \quad y(0) = 1.$

22. (a) Draw a direction field for the differential equation $2y' - y = e^{t/3}, \quad y(0) = a$. How do solutions appear to behave as t becomes large? Does the behavior depend on the choice of the initial value a? Let a_0 be the value for a for which the transition from one type of behavior to another occurs. Estimate the value of a_0.

(b) Solve the initial value problem and find the critical value a_0 exactly.

(c) Describe the behavior of the solution corresponding to the initial value a_0.