Mathematics 251: Lab 1 VECTOR CALCULUS

In this lab, we shall use Maple to compute dot and cross products of vectors, the length of a vector, the derivative of a vector function, and to plot vector functions. We shall also use Maple to compute the curvature of a space curve.

Please turn in only the printout of your Maple worksheet, which should include the Maple commands you input and Maple's response. Your worksheet should also include explicit answers to all questions asked and labels on any plots included in your worksheet. These should be inserted by using the **text** feature of Maple. Also use the **text** feature of Maple to include your name and section number before each problem (do NOT write any of this material in by hand). Be sure to remove from the worksheet any extraneous material and any errors you have made.

Before you start up Maple, first copy the file lab1.mws into your directory by using the command cp⊔/eden/u8/falk/math251/lab1.mws⊔lab1.mws
where the symbol ⊔ denotes a space. After you start up Maple, import this file into Maple by choosing **Open** from the **File** menu as described in the handout **Instructions for Use of Maple in Mathematics 251**. You will be asked to execute the Maple commands that have been placed in this worksheet and to add some of your own.

Note that in order to use Maple's commands for computing dot and cross products, it is first necessary to type the command `with(linalg):`. To access various plotting commands, one must also type the command `with(plots):`. These commands are already included in the file lab1.mws.

1a. Execute the following sequence of commands (included in lab1.mws) which illustrate how vectors are defined in Maple, how Maple computes dot and cross products and the length of a vector, and forms the product of a scalar with a vector.

```
# Dot and Cross Products and Norm
with(linalg):  with(plots):
v:= [1,2,3];
w:= [-1,1,2];
dpvw:= dotprod(v,w);
cpvw:= crossprod(v,w);
lenv:= sqrt(dotprod(v,v));  # compute the length of v
u := v/lenv; # produces a unit vector in the direction of v
w[1];   # gives the first component of w
```

Note that some of the Maple commands described above will be needed to do Problem 2.

2a. Execute the following sequence of Maple commands (included in lab1.mws), which illustrate how vector functions are defined, differentiated, evaluated, and plotted.

```
# Vector Functions -- Space Curves
restart;
with(linalg):  with(plots):
r:= [1-4*Pi^2*(cos(t)^2-sin(t)^2),cos(t),sin(t)];
v:= diff(r,t);    # differentiates r with respect to t
subs(t=1,r);      # evaluates r at t=1
spacecurve(r,t=-Pi..2*Pi); # Plots the space curve r
```

2b. With `v` defined as in 2a, compute the length of `v` (note how this was done in Problem 1a) and use the `simplify` command to simplify it.

2c. The following formulas for the unit tangent vector T, normal vector N, and the curvature κ can be found in your book:

$$\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}, \qquad \mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}, \qquad \kappa = \frac{|d\mathbf{T}/dt|}{|\mathbf{v}|}.$$

Use Maple to define expressions for each of these, using the letters given above, and then simplify these expressions as much as possible. Do not `restart` since you will need the expressions defined previously in parts 2a and 2b. Note that Maple often defines intermediate quantities (such as `%1, %2`) and then writes the desired expression using these quantities.

Maple does computations in the most general case, assuming that quantities may take on complex (imaginary) values (denoted by the letter `I`) as well as real values. To get Maple to give results which are much simpler by restricting quantities to real values, you must slightly modify some of the commands. In this problem, when you compute $|d\mathbf{T}/dt|$, use the `dotprod` command of the form `dotprod(u,v,'orthogonal');` which tells Maple that the vectors `u` and `v` are real. Also note that the result obtained by Maple for κ has an obvious further simplification. Rather than spending time figuring out how to get Maple to do it, leave this answer as is and proceed to the next part.

If you have done the computations correctly, you should obtain expressions which are equivalent (although they may not be identical) to the following:

$$|\mathbf{v}| = \sqrt{256\,\pi^4\cos^2(t) - 256\,\pi^4\cos^4(t) + 1},$$

$$T = \frac{[16\,\pi^2\cos(t)\sin(t), -\sin(t), \cos(t)]}{\sqrt{256\,\pi^4\cos^2(t) - 256\,\pi^4\cos^4(t) + 1}},$$

$$|d\mathbf{T}/dt| = \sqrt{\frac{768\,\pi^4\cos^4(t) - 768\,\pi^4\cos^2(t) + 256\,\pi^4 + 1}{(256\,\pi^4\cos^2(t) - 256\,\pi^4\cos^4(t) + 1)^2}},$$

$$\kappa = \sqrt{\frac{768\,\pi^4\cos^4(t) - 768\,\pi^4\cos^2(t) + 256\,\pi^4 + 1}{(256\,\pi^4\cos^2(t) - 256\,\pi^4\cos^4(t) + 1)^3}},$$

2d. As a further check of your result, use Maple to compute the curvature using the formula

$$\kappa_1 = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3},$$

where $\mathbf{a}$ is the acceleration vector $\mathbf{a} = d\mathbf{v}/dt$.

Note that Maple gives you an error message if you try to apply the simplify command to the cross product of two vectors. In 2d, this can be avoided by first computing $|\mathbf{v} \times \mathbf{a}|$ before simplifying.*

2e. Use Maple to verify that the vectors T and N are orthogonal at any value of t, i.e., verify that $T \cdot N = 0$. You may need to use the `simplify` command.

*The following information is not needed to do this assignment. It explains some peculiarities of Maple; in particular, why Maple gave you an error message if you tried to use `simplify` directly on the cross product of two vectors. Maple has another way of expressing vectors using the `vector` command, i.e., instead of defining a vector `v` by `v:= [1,2,3];`, an alternative notation is `v:= vector([1,2,3]);` When vectors are defined using the `vector` command, it is more complicated to work with them in Maple. For example, the `evalm` command must be used to do simple vector operations such as adding two vectors, i.e., `evalm(v+w)` or multiplying a vector by a scalar such as `evalm(v/lenv)`. Unfortunately, even if two vectors are defined without the `vector` command, the `crossprod` command produces a vector which is defined in terms of this command. Thus, to simplify a vector `B` defined using the `vector` command (or produced by the `crossprod` command), you must type `map(simplify,B);` . Just typing `simplify(B);` will produce an error message.