Math 251 Formula Sheet

Sections 16-18 Exam 1 October 8, 1998

$$\boldsymbol{a} =< a_1, a_2, a_3 >, \qquad \boldsymbol{b} =< b_1, b_2, b_3 >, \qquad \boldsymbol{a}\cdot\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$$

$$\text{Scalar projection of } \boldsymbol{b} \text{ onto } \boldsymbol{a} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|}, \qquad \text{Vector projection of } \boldsymbol{b} \text{ onto } \boldsymbol{a} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|^2}\boldsymbol{a}$$

$$\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\boldsymbol{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\boldsymbol{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\boldsymbol{k} = |\boldsymbol{a}||\boldsymbol{b}|\sin\theta$$

Line through (x_0, y_0, z_0) parallel to $\boldsymbol{v} =< a, b, c >$: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

Plane through $(x_0, y_0, z_0) \perp \boldsymbol{n} =< a, b, c >$: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$$\text{length of curve } C \text{ between } t = a \text{ and } t = b \qquad L = \int_a^b |\boldsymbol{r}'(t)|\,dt,$$

$$\text{unit tangent vector } \boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|}, \qquad \text{principal unit normal } \boldsymbol{N}(t) = \frac{\boldsymbol{T}'(t)}{|\boldsymbol{T}'(t)|},$$

$$\text{curvature } \kappa(t) = \frac{|\boldsymbol{T}'(t)|}{|\boldsymbol{r}'(t)|} = \frac{|\boldsymbol{r}'(t)\times\boldsymbol{r}''(t)|}{|\boldsymbol{r}'(t)|^3}$$

$$\text{acceleration} \qquad \boldsymbol{a} = \frac{d^2s}{dt^2}\boldsymbol{T} + \kappa\left(\frac{ds}{dt}\right)^2\boldsymbol{N}, \qquad \text{where} \qquad s = |\boldsymbol{v}|$$

$$\text{Total differential:} \qquad dz = f_x(x,y)dx + f_y(x,y)dy$$

$$\text{Chain rule:} \qquad z = f(x,y), \quad x = g(t), \quad y = h(t), \qquad \frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\text{Chain rule: } z = f(x,y), \quad x = g(s,t), \quad y = h(s,t)$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}, \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t},$$

$$\nabla f =< f_x, f_y, f_z >, \qquad D_u f(x,y) = \nabla f(x,y)\cdot u$$

Equation of tangent plane to surface $F(x,y,z) = 0$ at (x_0, y_0, z_0):
$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$

Second Derivative Test: If $f_x(a,b) = f_y(a,b) = 0$ and $D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$, then

i) If $D > 0$ and $f_{xx}(a,b) > 0$, local minimum at (a,b),
ii) If $D > 0$ and $f_{xx}(a,b) < 0$, local maximum at (a,b),
iii) If $D < 0$, saddle point at (a,b).