Math 321 Assignment 4 Computer Problem

Consider the nonlinear pendulum equation

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta - k\frac{d\theta}{dt}$$

in the case when $L = 1$, $g = 1$, $k = 3$. Then $k^2 > 4Lg$, so we are in the case of large friction.

Use *Matlab* to obtain two phase plane plots, one near the equilibrium position $\theta = 0$ and one near the equilibrium position $\theta = \pi$. For each, state whether the plot shows the equilibrium position is *stable* or *unstable*.

The following describes how *Matlab* can be used to draw the direction field and the phase plane for an autonomous second order differential equation, i.e., an equation of the form

$$\frac{d^2x}{dt^2} = f(x, dx/dt).$$

To solve a second order differential equation with *Matlab*, we first need to write it as a first order system, i.e., let $\vec{y} = (y_1, y_2) = (x, dx/dt)$. Then we have

$$\frac{dy_1}{dt} = y_2, \qquad \frac{dy_2}{dt} = f(y_1, y_2).$$

In vector notation, our equation is now

$$\frac{d\vec{y}}{dt} = \vec{F}(\vec{y}), \qquad \vec{F} = (F_1, F_2) = (y_2, f(y_1, y_2)).$$

A *Matlab* command that can be used to plot a direction field is the command

```
quiver(x1,x2,u1,u2)
```

which displays vectors as components `(u1,u2)` at the points `(x1,x2)`.

In our case, we want to display the vector $(y_2, f(y_1, y_2))$ at the point (y_1, y_2). To relate this notation to what is done in the textbook, we set $v = y_2$. Then, plotting a short straight line with slope $f(x, v)/v$ at the point (x, v) is equivalent to plotting the vector $(y_2, f(y_1, y_2)) = (v, f(x, v))$ at the point $(y_1, y_2) = (x, v)$ in the phase plane.

Setting $\vec{y} = (\theta, d\theta/dt)$, we can define the right hand side F of the nonlinear pendulum equation by the *Matlab* inline function:

```
F = inline('[y(2);-sin(y(1))-3*y(2)]','t','y')
```

where the symbol `'` changes row vectors to column vectors.

To plot the direction field, we first define the set of points at which we want to display the vectors. For example,

```
y1 = -2:.25:2  % computes the points -2, -1.75, ..., 2
y2 = -2:.25:2
```

Next, we compute the vectors we want to plot. Note, we need to compute a vector for each point in our grid.

```
n1=length(y1);  % computes the length of the vector y1
n2=length(y2);  % computes the length of the vector y2
u1=zeros(n2,n1);  % initializes u1, the first component of our vector
u2=zeros(n2,n1);  % initializes u2, the first component of our vector
for i=1:n1
  for j=1:n2
    u = feval(F,0,[y1(i);y2(j)]);  % evaluates F at at t=0, y=[y1(i);y2(j)]
    u1(j,i) = u(1);
    u2(j,i) = u(2);
  end
end
```

Then, we get the direction field by using the function `quiver`, i.e.,

```
quiver(y1,y2,u1,u2);
```

To plot some solution curves in the same window, we first use the command

```
hold on
```

We can then use the command `ode45` to obtain the solution of the differential equation for a given set of initial conditions and the command `plot` to plot them. For example, a set of solution curves around the equilibrium position 0 can be obtained by the commands

```
for y10=-1:.5:1
for y20=-2:.5:2
  [ts,ys] = ode45(F,[0,10],[y10;y20]);
  plot(ys(:,1),ys(:,2))
end
end
hold off
```

Execute all these commands in *Matlab* and print out and hand in the resulting plot. Then change the initial conditions to obtain a similar plot near the equilibrium position $\theta = \pi$.