Math 373 Assignment 10 (Due Tuesday, December 9)

1. Consider the approximation of the initial value problem $y' = f(x, y), \qquad y(x_0) = y_0$, by a class of explicit linear multistep methods of the form:

$$y_{n+1} = a_0 y_n + a_1 y_{n-1} + h[b_0 f_n + b_1 f_{n-1}].$$

If the constant $a_0$ is considered fixed, determine values of the constants $a_1$, $b_0$, and $b_1$ (expressed in terms of $a_0$), such that the local truncation error of the method will be $O(h^3)$.

2. Determine which of the following methods are convergent. State a reason for your conclusion.

i) $y_{n+1} = y_n + hf_{n+1}$,

ii) $y_{n+1} = \frac{3}{2}y_n - \frac{1}{2}y_{n-1} + hf_n$,

iii) $y_{n+1} = 3y_n - 2y_{n-1} + h[f_{n+1} - 2f_n]$.

3. We wish to determine the interval of absolute stability for the method:

$$y_{n+1} = y_n + \frac{h}{2}[3f_n - f_{n-1}].$$

a. First show that the interval of absolute stability are the values of $h\lambda$ satisfying

$$-1 \leq \frac{[1 + 3h\lambda/2]}{2} \pm \frac{\sqrt{1 + h\lambda + 9h^2\lambda^2/4}}{2} \leq 1.$$

b. Next show that this is equivalent to the two inequalities

$$\sqrt{1 + h\lambda + 9h^2\lambda^2/4} \leq 1 - 3h\lambda/2,$$
$$\sqrt{1 + h\lambda + 9h^2\lambda^2/4} \leq 3 + 3h\lambda/2.$$

c. Since the left hand side of these inequalities is positive, we at least need the right hand side to be positive, which requires that $-2 \leq h\lambda \leq 2/3$. Under these conditions, we can then solve these inequalities by squaring both sides to determine further restrictions on the values of $h\lambda$ that give absolute stability.