

Math 373 Assignment 3 (Due Thursday, September 25.)

Page 115: 5a (only for degrees one and two), 9a, 19d (Do this for $n = 2$ instead of $n = 3$, using the points $x_0 = 0, x_1 = .25, x_2 = 1.0$).

Page 127: 7, 11b(assume part (a) is true), 17

Computer Problem: Before beginning this problem, copy the file <http://www.math.rutgers.edu/~falk/math373/interppoly.m> to the directory you will be using when you start up Matlab.

a: Let $[a, b] = [-5, 5]$. For each $n = 4, 8, 16$, and 32 , plot over the interval $[a, b]$ the error function $e(x) = f(x) - P_n(x)$, where $f(x) = 1/(1+x^2)$ and $P_n(x)$ is the polynomial of degree $\leq n$ which interpolates f at the equally spaced interpolation points $x_i = a + i(b - a)/n$, $i = 0 \dots n$. Record the approximate maximum of $|e(x)|$ (as seen from the graph) for each n and approximately where it occurs. To get you started, the case $n = 4$ can be done by typing (or cutting and pasting) the following commands into Matlab

```
% Problem 3a
a=-5; b=5;
n=4;
xin=linspace(a,b,n+1);
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,ytrue-yout)
```

The other values of n can be done by by entering a new value of n and then re-entering the other commands.

b: Repeat part (a), this time using the polynomial $Q_n(x)$ of degree $\leq n$ which interpolates f at the (Chebyshev) points

$$x_i = \frac{a+b}{2} + \frac{b-a}{2} \cos\left(\frac{(2i+1)\pi}{2n+2}\right), \quad i = 0, \dots, n.$$

To get you started, the case $n = 4$ can be done by typing (or cutting and pasting) the following commands into Matlab.

```
% Problem 3b
close all % closes plotting windows
a=-5; b=5;
n=4;
x=linspace(1,2*n+1,n+1);
```

2

```
xin = (a+b)/2 + (b-a)/2 * cos(x*pi/(2*n+2));  
yin= 1./(1+xin.^2);  
xout=linspace(a,b,100);  
[yout, cof] = interppoly(xin,yin,xout);  
ytrue= 1./(1+xout.^2);  
plot(xout,ytrue-yout)
```

c. Based on your plots, does the choice of interpolation points make a difference in the error in the approximation? Which choice is better in this case?