Math 373 Assignment 9 (Due Tuesday, November 25)

1a. Use the modified Euler's method with step size $h = 1/2$ to compute approximations to $y(1/2)$ and $y(1)$, where $y(x)$ is the solution of the Initial Value Problem

$$y' = 1 - 8xy, \qquad y(0) = 0.$$

1b. Repeat part (a) using Heun's method.

2. Let $Y = \begin{pmatrix} w \\ z \end{pmatrix}$ and $F(x, Y) = \begin{pmatrix} f_1(x, w, z) \\ f_2(x, w, z) \end{pmatrix}$.

2a. Euler's method for the system of differential equations $Y' = F(x, Y)$, with initial condition $Y(x_0) = Y_0$, is given by:

$$Y_{n+1} = Y_n + hF(x_n, Y_n).$$

Find approximations to $w(h)$ and $z(h)$ for the system

$$w' = z, \quad z' = -cw, \qquad w(0) = a, \quad z(0) = b,$$

where a, b, c are given constants.

2b. Heun's method for the system of differential equations $Y' = F(x, Y)$, with initial condition $Y(x_0) = Y_0$, is given by:

$$Y_{n+1} = Y_n + \frac{h}{2}F(x_n, Y_n) + \frac{h}{2}F(x_n + h, Y_n + hF(x_n, Y_n)).$$

Find approximations to $w(h)$ and $z(h)$ for the system of part (a).

3. In this problem, we use *Matlab*'s built in codes `ode23` and `ode23s` (the `s` means the code is appropriate for "stiff" systems) to solve the initial value problem for the van der Pol equation (with parameter $\mu > 0$):

$$y'' - \mu(1 - y^2)y' + y = 0, \qquad y(0) = 2, \quad y'(0) = 0.$$

To do so, let $y_1 = y$, $y_2 = y_1'$ and write this second order equation as the first order system

$$y_1' = y_2, \qquad y_2' = \mu(1 - y_1^2)y_2 - y_1, \qquad y_1(0) = 2, \quad y_2(0) = 0.$$

3a. When $\mu = 1$, find an approximate solution on the interval $[0, 20]$ using the *Matlab* code `ode23s` and plot the first component $y_1 = y$. This may be done by executing the following commands:

```
clear
t0 = 0
tfinal = 20
tspan = [t0 tfinal]
y0 = [2 0]'
mu = 1
vdpfcn = @(t,y)[ y(2,:); (mu*(1-y(1,:).^2).*y(2,:) - y(1,:))];
```

```
options = odeset('RelTol',1e-2,'AbsTol',1e-5);
[t y] = ode23s(vdpfcn,tspan,y0,options);
plot(t,y(:,1),'o')
title('{\mu=1}');     % Use this statement to place a title on your graph
grid on               % Use this statement to see the values more clearly
```

3b. Repeat part (a) replacing `ode23s` by `ode23`.

3c. Repeat part (a) for the case `mu = 1000`. For this case, also set `tfinal = 3000`.

3d. Try to do part (c) using the code `ode23`. If the code runs longer than one minute, abort the computation by holding down the **Control** key and typing **c**. When `mu =1000`, this is a stiff system, for which standard codes often have problems.

Hand in the plots for parts (a), (b), (c), making sure each is labeled with the appropriate value of `mu`.

The motion is periodic. My looking at the plots, determine a rough estimate of the period for each problem, and write it on your plots.