A REMARK ON SCHUBERT CELLS AND DUALITY OF ORBITS ON FLAG MANIFOLDS

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Abstract. It is known that the closure of an arbitrary $K_{\mathbb{C}}$-orbit on a flag manifold is expressed as a product of a closed $K_{\mathbb{C}}$-orbit and a Schubert cell ([M2], [Sp]). We already applied this fact to the duality of orbits on flag manifolds ([GM]). We refine here this result and give its new applications to the study of domains arising from the duality.

1. Duality of orbits on flag manifolds

Let $G_{\mathbb{C}}$ be a connected complex semisimple Lie group and $G_{\mathbb{R}}$ a connected real form of $G_{\mathbb{C}}$. Let $K_{\mathbb{C}}$ be the complexification in $G_{\mathbb{C}}$ of a maximal compact subgroup K of $G_{\mathbb{R}}$. Let $X = G_{\mathbb{C}}/P$ be a flag manifold of $G_{\mathbb{C}}$ where P is an arbitrary parabolic subgroup of $G_{\mathbb{C}}$. Then there exists a natural one-to-one correspondence between the set of $K_{\mathbb{C}}$-orbits S and the set of $G_{\mathbb{R}}$-orbits S' on X given by the condition:

$$\text{(A)} \qquad S \leftrightarrow S' \iff S \cap S' \text{ is non-empty and compact}$$

([M3]). In the following, we will identify orbits S with $K_{\mathbb{C}}$-P double cosets and S' with $G_{\mathbb{R}}$-P cosets.

We defined in [GM] a subset $C(S)$ of $G_{\mathbb{C}}$ by

$$C(S) = \{x \in G_{\mathbb{C}} \mid xS \cap S' \text{ is non-empty and compact in } X = G_{\mathbb{C}}/P\}$$

where S' is the $G_{\mathbb{R}}$-orbit on X given by (A).

If S is closed, then S' is open ([M1]) and so the condition

$$xS \cap S' \text{ is non-empty and compact in } G_{\mathbb{C}}/P$$

implies

$$xS \subset S'.$$

Hence the set $C(S)_0$ is the cycle domain (cycle space) for S' ([WW]) where $C(S)_0$ denote the connected component of $C(S)$ containing the identity.

On the other hand, let S_0 denote the unique open $K_{\mathbb{C}}$-B double coset in $G_{\mathbb{C}}$ where B is a Borel subgroup of $G_{\mathbb{C}}$ contained in P. (We will keep this notation for the whole note.) Then S_0' is the only one closed $G_{\mathbb{R}}$-B double coset in $G_{\mathbb{C}}$ and the condition

$$xS_0 \cap S_0' \text{ is non-empty and compact in } G_{\mathbb{C}}/B$$

implies

$$xS_0 \supset S_0'.$$

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Let $\{S_j \mid j \in J\}$ be the set of $K_{\mathbb{C}}$-B double cosets in $G_{\mathbb{C}}$ of codimension one and $T_j = S_j^{cl}$ denote the closure of S_j. The sets T_j will play an important role in our constructions.

The compliment of S_0 in $G_{\mathbb{C}}$ is written as

$$\bigcup_{j \in J} T_j$$

(by Theorem 2 in Section 2). So the set $C(S_0)$ is the compliment of the infinite family of complex hypersurfaces

$$T_j^{-1} g \quad (j \in J,\ g \in S_0')$$

and hence the connected component $C(S_0)_0$ is Stein.

This domain is sometimes called the "Iwasawa domain" since it is a maximal domain where all Iwasawa decompositions can be holomorphically extended from $G_{\mathbb{R}}$.

In [GM], we defined

$$C = \bigcap C(S)$$

where we take the intersection for all $K_{\mathbb{C}}$-orbits on X on all flag manifolds $X = G_{\mathbb{C}}/P$ of $G_{\mathbb{C}}$ and conjectured

$$C = \widetilde{D_0} Z$$

(Conjecture 1.3) where $D_0 = \widetilde{D_0}/K_{\mathbb{C}}$ is the domain introduced by [AG] (which is sometimes denoted as Ω_{AG}) and Z is the center of $G_{\mathbb{C}}$. For connected components, it means

$$C_0 = \widetilde{D_0}. \tag{B}$$

It is proved in Proposition 8.3 of [GM] that

$$C_0 = C(S_0)_0.$$

In other words, $C(S)_0$ is minimal when $S = S_0$. We believe that it is one of central facts of this theory since it gives a very strong estimate of all $C(S)$ through $C(S_0)$ only. So the conjecture (B) is equivalent to

$$C(S_0)_0 = \widetilde{D_0}.$$

Recently, the inclusion $C(S_0)_0 \subset \widetilde{D_0}$ is proved by Barchini ([B]). On the other hand, the opposite inclusion $C(S_0)_0 \supset \widetilde{D_0}$ is proved for all classical cases ([GM], [KS]) and exceptional Hermitian cases ([GM]). It is announced that it is proved for all spaces in [H]. (We had no chance to see this preprint.)

Remark 1. In [FH], the authors deduce the inclusion $C_0 \subset \widetilde{D_0}$ from their result about $C(S)$ for closed S and Proposition 8.1 in [GM]. As we showed above, this inclusion is already the consequence of Proposition 8.3 in [GM] and [B]. So it does not need the results in [FH].

2. Schubert cells in the category of $K_{\mathbb{C}}$-B double cosets

The principal idea of our considerations in [GM] was that $C(S)_0$ will be essentially independent of neither S nor the flag manifold $X = G_{\mathbb{C}}/P$. To justify it, we need to build bridges between $C(S)$ for different S and for it we need to see connections between different $K_{\mathbb{C}}$-orbits. It turns out that Schubert cells are very efficient tool for such considerations as in Section 2 and Section 8 in [GM]. They give a possibility to obtain an important information about general $C(S)$ from a consideration of simplest S. Here we refine connections between $K_{\mathbb{C}}$-orbits and Schubert cells and give more examples of applications.

For a simple root α in the root system with respect to the order defined by B, we can define a parabolic subgroup

$$P_\alpha = B \cup Bw_\alpha B$$

of $G_{\mathbb{C}}$ such that $\dim_{\mathbb{C}} P_\alpha = \dim_{\mathbb{C}} B + 1$.

Lemma 1. *Let S_1 be a $K_{\mathbb{C}}$-B double coset. Then we have*:

(i) *If $\dim_{\mathbb{C}} S_1 P_\alpha = \dim_{\mathbb{C}} S_1$, then $S_1^{cl} P_\alpha = S_1^{cl}$.*

(ii) *If $\dim_{\mathbb{C}} S_1 P_\alpha = \dim_{\mathbb{C}} S_1 + 1$, then there exists a $K_{\mathbb{C}}$-B double coset S_2 such that $S_1^{cl} P_\alpha = S_2^{cl}$.*

Proof. Though this lemma follows easily from [M2] Lemma 3, we will give a proof for the sake of completeness. Write $S_1 = K_{\mathbb{C}} gB$. Then we have a natural bijection

$$(g^{-1}K_{\mathbb{C}} g \cap P_\alpha)\backslash P_\alpha/B \cong K_{\mathbb{C}}\backslash K_{\mathbb{C}} gP_\alpha/B = K_{\mathbb{C}}\backslash S_1 P_\alpha/B$$

by the map $x \mapsto gx$.

(i) If $\dim_{\mathbb{C}} S_1 P_\alpha = \dim_{\mathbb{C}} S_1$, then $(g^{-1}K_{\mathbb{C}} g \cap P_\alpha)B/B$ is Zariski open in $P_\alpha/B = P^1(\mathbb{C})$ and hence it is dense. So we have

$$S_1^{cl} = (K_{\mathbb{C}} gB)^{cl} \supset S_1 P_\alpha \supset S_1$$

and therefore $S_1^{cl} = S_1^{cl} P_\alpha$.

(ii) Suppose $\dim_{\mathbb{C}} S_1 P_\alpha = \dim_{\mathbb{C}} S_1 + 1$. Then there exists a $p \in P_\alpha$ such that $(g^{-1}K_{\mathbb{C}} g \cap P_\alpha)pB/B$ is Zariski open in $P_\alpha/B = P^1(\mathbb{C})$ since the number of $K_{\mathbb{C}}$-B double cosets in $G_{\mathbb{C}}$ is finite. If we write $S_2 = K_{\mathbb{C}} gpB$, then we have

$$(S_2)^{cl} \supset S_1 P_\alpha \supset S_2$$

and therefore $S_2^{cl} = S_1^{cl} P_\alpha$. q.e.d.

Theorem 1. *Let S_1 be a $K_{\mathbb{C}}$-B double coset in $G_{\mathbb{C}}$ and w an element of W. Then we have*:

(i) *$S_1^{cl}(BwB)^{cl} = S_2^{cl}$ for some $K_{\mathbb{C}}$-B double coset S_2.*

(ii) (*minimal expression*) *There exists a $w' \in W$ such that $w' < w$ (Bruhat order), $\ell(w') = \dim_{\mathbb{C}} S_2 - \dim_{\mathbb{C}} S_1$ and that*

$$S_1^{cl}(Bw'B)^{cl} = S_2^{cl}.$$

Here $\ell(w') = \dim_{\mathbb{C}} Bw'B - \dim_{\mathbb{C}} B$ is the length of w'.

Proof. (i) This follows from Lemma 1 because every Schubert cell $(BwB)^{cl}$ is written as

$$(BwB)^{cl} = P_{\alpha_1} \cdots P_{\alpha_\ell}$$

where $w = w_{\alpha_1} \cdots w_{\alpha_\ell}$ is a minimal expression of $w \in W$.

(ii) By Lemma 1, we can choose a subsequence $\beta_1, \ldots, \beta_q$ ($q = \dim_{\mathbb{C}} S_2 - \dim_{\mathbb{C}} S_1$) of $\alpha_1, \ldots, \alpha_\ell$ such that

$$\dim_{\mathbb{C}} S_1^{cl} P_{\beta_1} \cdots P_{\beta_k} = \dim_{\mathbb{C}} S_1^{cl} P_{\beta_1} \cdots P_{\beta_{k-1}} + 1$$

for $k = 1, \ldots, q$ and that

$$S_2^{cl} = S_1^{cl}(BwB)^{cl} = S_1^{cl} P_{\alpha_1} \cdots P_{\alpha_\ell} = S_1^{cl} P_{\beta_1} \cdots P_{\beta_q} = S_1^{cl}(Bw'B)^{cl}$$

with $w' = w_{\beta_1} \cdots w_{\beta_q}$. q.e.d.

Remark 2. $S_1^{cl}(BwB)^{cl} = S_2^{cl}$ implies $S_1^{cl} \subset S_2^{cl}$. But $S_1^{cl} \subset S_2^{cl}$ does not always imply $S_1^{cl}(BwB)^{cl} = S_2^{cl}$ for some w (c.f. [M2]).

Definition 1. For every $K_{\mathbb{C}}$-B double coset S, we can define, by Theorem 1, a subset $J(S)$ of J by

$$J(S) = \{j \in J \mid S^{cl}(BwB)^{cl} = T_j \text{ for some } w \in W\}.$$

Lemma 2. *Let S be a non-open $K_{\mathbb{C}}$-B double coset. Then there exists a simple root α such that*

$$\dim_{\mathbb{C}} SP_\alpha = \dim_{\mathbb{C}} S + 1.$$

Proof. Write $G_{\mathbb{C}} = (Bw_0B)^{cl} = P_{\alpha_1} \cdots P_{\alpha_m}$ with the longest element w_0 in W. If

$$\dim_{\mathbb{C}} SP_\alpha = \dim_{\mathbb{C}} S$$

for all simple roots α, then we have, by Lemma 1,

$$G_{\mathbb{C}} = S^{cl} G_{\mathbb{C}} = S^{cl} P_{\alpha_1} \cdots P_{\alpha_m} = S^{cl},$$

a contradiction. q.e.d.

Theorem 2. *If $\ell(w) < \mathrm{codim}_{\mathbb{C}} S$, then*

$$S^{cl}(BwB)^{cl} \subset T_j$$

for some $j \in J(S)$.

Proof. Since $\mathrm{codim}_{\mathbb{C}} S^{cl}(BwB)^{cl} = d > 0$, we can choose simple roots $\alpha_1, \ldots, \alpha_{d-1}$ such that

$$\mathrm{codim}_{\mathbb{C}} S^{cl}(BwB)^{cl} P_{\alpha_1} \cdots P_{\alpha_{d-1}} = 1$$

by Lemma 2. Since $(BwB)^{cl} P_{\alpha_1} \cdots P_{\alpha_{d-1}} = (Bw'B)^{cl}$ for some $w' \in W$, we have

$$S^{cl}(BwB)^{cl} \subset S^{cl}(Bw'B)^{cl} = T_j$$

for some $j \in J(S)$. q.e.d.

3. Applications

Definition 2. For every subset J' in J, we define a domain $\Omega(J')$ in $G_{\mathbb{C}}$ by

$$\Omega(J') = \{x \in G_{\mathbb{C}} \mid xT_j \cap S_0' = \phi \text{ for all } j \in J'\}_0.$$

We can prove the following corollary:

Corollary *Let S be a closed $K_{\mathbb{C}}$-P double coset in $G_{\mathbb{C}}$. Write $S = S_1^{cl}$ with the dense $K_{\mathbb{C}}$-B double coset S_1 in S. Then we have*

$$C(S)_0 = \Omega(J(S_1)).$$

Remark 3. (i) We can see $C(S_0)_0 = \Omega(J)$. By the same argument as for $C(S_0)_0$ in Section 1, we can prove $\Omega(J')$ is Stein for every subset J' in J. So the Steinness of $C(S)_0$ ([W]) becomes a corollary of this equivalence $C(S)_0 = \Omega(J(S_1))$ (c.f. [HW]).

(ii) It is clear that $\Omega(J') \supset \Omega(J)$ for every subset J' in J. So we have

$$C(S)_0 \supset C(S_0)_0.$$

But this inclusion was already proved in Proposition 8.3 in [GM]. This is natural because the way of proof of the corollary below is essentially the same as that of Proposition 8.3 in [GM]. So the above corollary may be considered as its refinement.

Proof of Corollary. Let x be an element on the boundary of $C(S)_0$. Then

$$xS \cap S_2'P \neq \phi$$

for some $G_{\mathbb{R}}$-P double coset $S_2'P$ in the boundary of S'. Here we take S_2 as the dense $K_{\mathbb{C}}$-B double coset contained in S_2P. Since S is right P-invariant, we have

$$xS \cap S_2' \neq \phi \quad \text{and} \quad \dim_{\mathbb{C}} S_2 > \dim_{\mathbb{C}} S.$$

Applying Theorem 1 (ii) to the pair $(S_2^{cl},\ G_{\mathbb{C}})$, we can take a $w \in W$ such that $\ell(w) = \operatorname{codim}_{\mathbb{C}} S_2$ and that

$$S_2(BwB)^{cl} = G_{\mathbb{C}}.$$

So we have $S_2(BwB)^{cl} \supset S_0$ and hence

$$S_2' \subset S_0'(Bw^{-1}B)^{cl}.$$

Since $xS \cap S_2' \neq \phi$, we have

$$xS \cap S_0'(Bw^{-1}B)^{cl} \neq \phi.$$

Hence

$$xS(BwB)^{cl} \cap S_0' \neq \phi$$

which implies $xT_j \cap S_0' \neq \phi$ for some $j \in J(S_1)$ by Theorem 2. Thus $x \notin \Omega(J(S_1))$.

Conversely, suppose

$$xT_j \cap S_0' \neq \phi$$

for some $T_j = S(BwB)^{cl} = S_1^{cl}(BwB)^{cl}$. Note that $j \in J(S_1)$ by Definition 1 and that we may assume $\ell(w) = \operatorname{codim}_{\mathbb{C}} S - 1 = \operatorname{codim}_{\mathbb{C}} S_1 - 1$ by Theorem 1 (ii). Then we have

$$xS \cap S_0'(Bw^{-1}B)^{cl} \neq \phi$$

and hence

$$xS \cap S_3' \neq \phi$$

for some $K_{\mathbb{C}}$-B double coset S_3 such that $S_3' \subset S_0'(Bw^{-1}B)^{cl}$. Hence $S_3(BwB)^{cl} \supset S_0$ and therefore $\dim_{\mathbb{C}} S_3 \geq \dim_{\mathbb{C}} G_{\mathbb{C}} - \ell(w) > \dim_{\mathbb{C}} S$. So we have

$$S_3' \cap S' = \phi$$

because S' is the union of $G_{\mathbb{R}}$-B double cosets S_4' satisfying $S_4 \subset S$. Hence we have

$$xS \not\subset S'$$

and therefore

$$x \notin C(S). \qquad \text{q.e.d.}$$

Remark 4. (i) The condition $\ell(w) = \mathrm{codim}_{\mathbb{C}} S - 1$ does "not always" imply

$$\mathrm{codim}_{\mathbb{C}}\, S^{cl}(BwB)^{cl} = 1.$$

Counter examples exist already for $G_{\mathbb{R}} = SU(2,1)$.

(ii) The construction of the domain $\Omega(J(S_1))$ is essentially equivalent to the construction of "Schubert domain" in [HW]. Unfortunately, their basic definition needs a correction and after this correction their proof of Corollary 3.2 corresponding to our Corollary is not complete. We can see that the proof of Corollary using the results in Section 2 is extremely simple. Let us explain the connection between these two constructions introducing notations in [HW].

Take a Borel subgroup B_0 of $G_{\mathbb{C}}$ so that $G_{\mathbb{R}}B_0$ is closed in $G_{\mathbb{C}}$. A Borel subgroup B of $G_{\mathbb{C}}$ is called an "Iwasawa Borel subgroup" if

$$B = g_0 B_0 g_0^{-1} \quad \text{for some } g_0 \in G_{\mathbb{R}}.$$

Let $Z = G_{\mathbb{C}}/Q$ be a flag manifold. Then we can take Q so that $Q \supset B_0$. Every Schubert cell Y in Z for B is written as

$$Y = (Bg_0wQ)^{cl} = (g_0B_0wQ)^{cl}$$

with some $w \in W$. Let C_0 be a closed $K_{\mathbb{C}}$-Q double coset (do not miss with C_0 of Section 1!). The "incidence divisor" H_Y is written as

$$H_Y = \{g \mid gC_0 \cap Y \neq \phi\} = YC_0^{-1} = (g_0B_0wQ)^{cl}C_0^{-1} = (C_0(Qw^{-1}B)^{cl}g_0^{-1})^{-1}.$$

In this point, in [HW], it is written: "If $\mathrm{codim}\, Y \leq \dim C_0 + 1$, then H_Y is a hypersurface in $G_{\mathbb{C}}$." It is wrong as we remarked in (i).

But if $\mathrm{codim}\, H_Y = 1$, then

$$H_Y^{-1} = C_0(Qw^{-1}B_0)^{cl}g_0^{-1} = T_j g_0^{-1}$$

for some $j \in J' = J(C_1)$ (where C_1 is the dense $K_{\mathbb{C}}$-B_0 double coset in C_0) and $g_0 \in G_{\mathbb{R}}$ by our notation.

So their definition of $y(D)$ should be corrected to

$$y(D) = \{Y = (g_0B_0wQ)^{cl} \mid \mathrm{codim}\, H_Y = 1\}.$$

Here D is the $G_{\mathbb{R}}$-orbit dual to C_0. (The condition $Y \subset Z \setminus D$ follows from $\operatorname{codim} H_Y = 1$ because

$$\begin{aligned} Y \cap D = \phi &\Longleftrightarrow DY^{-1} = D(Qw^{-1}B_0)^{cl} g_0^{-1} \not\ni e \\ &\Longleftrightarrow D(Qw^{-1}B_0)^{cl} \not\ni g_0 \\ &\Longleftrightarrow D(Qw^{-1}B_0)^{cl} \cap G_{\mathbb{R}} B_0 = \phi \\ &\Longleftrightarrow C_0(Qw^{-1}B_0)^{cl} \cap K_{\mathbb{C}} B_0 = \phi \\ &\Longleftrightarrow \operatorname{codim} C_0(Qw^{-1}B_0)^{cl} \geq 1.) \end{aligned}$$

The Schubert domain is defined as

$$\Omega_S(D) = \left\{ G_{\mathbb{C}} \setminus \left(\bigcup_{Y \in y(D)} H_Y \right) \right\}_0 .$$

This definition is equivalent to ours because

$$\begin{aligned} g \notin \bigcup_{Y \in y(D)} H_Y &\Longleftrightarrow g^{-1} \notin T_j g_0^{-1} \text{ for all } j \in J' \text{ and } g_0 \in G_{\mathbb{R}} \\ &\Longleftrightarrow g^{-1} G_{\mathbb{R}} B_0 \cap T_j = \phi \text{ for all } j \in J' \\ &\Longleftrightarrow G_{\mathbb{R}} B_0 \cap g T_j = \phi \text{ for all } j \in J'. \end{aligned}$$

(iii) So in their Corollary 3.2 in [HW], (2) $\operatorname{codim}_Z(Y) = q + 1$ does not imply (3) H_Y is a hypersurface in Ω. Thus the proof of Corollary 3.2 is incomplete.

Remark 5. The problem of the description of the domain of cycles $C(S)_0$ for groups $G_{\mathbb{R}}$ of Hermitian type is simpler than the general case. Firstly, in this case, $D_0 = \widetilde{D_0}/K_{\mathbb{C}}$ has a very simple description: $D_0 \cong G_{\mathbb{R}}/K \times \overline{G_{\mathbb{R}}/K}$ (Proposition 2.2 in [GM]). As usual, the equality $C(S)_0 = \widetilde{D_0}$ for S ($\leftrightarrow S'$) of nonholomorphic type is reduced to two inclusions. The proof of $C(S)_0 \subset \widetilde{D_0}$ in [WZ1] had a mistake which was corrected in [WZ2]. The opposite inclusion was checked in [WZ1] for classical Hermitian groups. In Proposition 2.4 of [GM], we gave a very simple proof of this inclusion for arbitrary groups of Hermitian type which is free of case-by-case considerations: the use of Schubert cells makes this fact almost trivial. The note [WZ2] also contains this fact with a proof referred to [HW] (which is incomplete as is explained in Remark 4) but without an appropriate reference on the preceding proof in [GM]. Moreover it asserts a misleading statement that the preprint [GM] does not contain a direct proof.

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