## Math review for the final exam

**Exam conditions** The final exam will be "open book, open notes". You may bring to the exam anything additional to help you which is *non-living*.\*

## Rules

- In each case, you should show your work, explain your answer, etc.: an answer alone generally will *not* be sufficient for full credit!
- Write answers to "mod" questions in standard form: although  $20 = 2 = -16 \mod 18$  etc., the standard form would be **2**: always an integer between 0 and the modulus.

**Problem 1** Which of  $(10^{44})^{55}$  and  $(10^{44}) \cdot (10^{55})$  is larger?

**Problem 2** What is 56 mod 6?

**Problem 3** What is 3798107541 mod 9?

**Problem 4** What is 3798107541 mod 11?

**Problem 5** What is  $7^{322222224512} \mod 8$ ? (7 = 8 - 1.)

**Problem 6** What is  $7^{322222224512} \mod 7$ ?

**Problem 7** What is  $7^{322222224512} \mod 6$ ? (7 = 6 + 1).

**Problem 8** What is  $3 \cdot (6^{200}) + 2 \cdot (8^{333}) + 4 \mod 7$ ?  $(6 = \dots \text{ and } 8 = \dots)$ 

**Problem 9** What is  $42^{1234567} \mod 4$ ?

**Problem 10** What is the table of addition mod 5?

**Problem 11** What is the table of multiplication mod 5?

**Problem 12** Solve  $2x + 1 = 4 \mod 5$ .

**Problem 13** Solve  $2x + 1 = 4 \mod 6$ .

**Problem 14** Solve  $3x + 1 = 4 \mod 6$ .

**Problem 15** Define **prime number**. Explain briefly why arithmetic mod a prime number is more like standard arithmetic than arithmetic mod a non-prime number.

**Problem 16** What is 23 in binary?

**Problem 17** What is 10110001 in decimal?

**Problem 18** How many different bitstrings 7 bits long are there?

**Problem 19** What is the xor of the bitstrings 11001 11100 and 10101 00111?

**Problem 20** Which is more work ("work"=one-digit arithmetic operations): squaring one hundred numbers 50 decimal digits long or adding ten pairs of numbers 5,000 decimal digits long?

<sup>\*</sup> The instructor will judge if something is "alive". I included this stipulation because of the following story: At Caltech one semester, a physics instructor said that on the final exam, students could use "anything they could carry in." One student carried in Richard Feynman. You should know who Richard Feynman is (ground-breaking theoretical physicist) and something of his character (first-class crazy person) to appreciate this.