1 A

(20) 1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x + 2}$

b) $\lim\limits_{x \to 0} \dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$

c) $\lim\limits_{x \to 1^-} \dfrac{3x + \ln x}{x}$

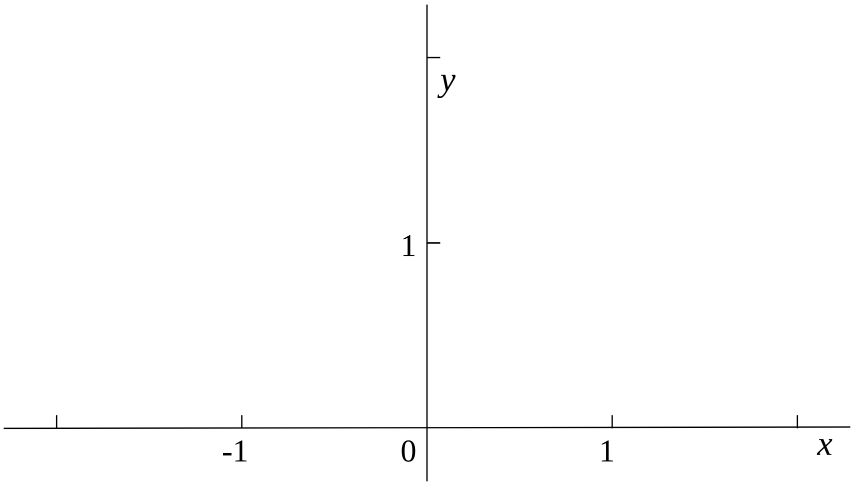
d) $\lim\limits_{x \to 0^+} \dfrac{2}{\sin x}$

(14) 2. Suppose that the function F is described by

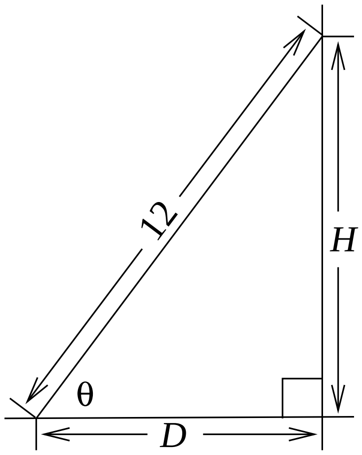
$$F(x) = \begin{cases} \cos x & \text{if } -\frac{\pi}{2} \le x \le 0 \\ Ax + B & \text{if } 0 < x \le 1 \\ x - 1 & \text{if } 1 < x \le 2 \end{cases} .$$

a) Find A and B so that F is continuous for all numbers in its domain. Briefly explain your answer.

b) Graph $y = F(x)$ on the axes given for the values of A and B found in a).



(12) 3. A ladder which is 12 feet long has one end on flat ground and the other end on the vertical wall of a building. H is the height from the ground to the point at which the ladder touches the building. D is the distance between the bottom of the ladder and the bottom of the wall. θ is the acute angle between the ladder and the ground.



a) Write H as a function of D: that is, a formula involving D and no other variable. What is the domain of this function?

b) Write H as a function of θ: that is, a formula involving θ and no other variable. What is the domain of this function?

(12) 4. a) Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \dfrac{2}{x+3}$.

b) Use your answer to a) to find an equation for the line tangent to the curve $y = \dfrac{2}{x+3}$ when $x = 2$.

(9) 5. Do *not* algebraically simplify the answers in this problem!

a) If $f(x) = -8x^7 + 4x^4 + 24$, what is $f'(x)$?

b) If $g(x) = \dfrac{x^2+1}{2e^x - 3}$, what is $g'(x)$?

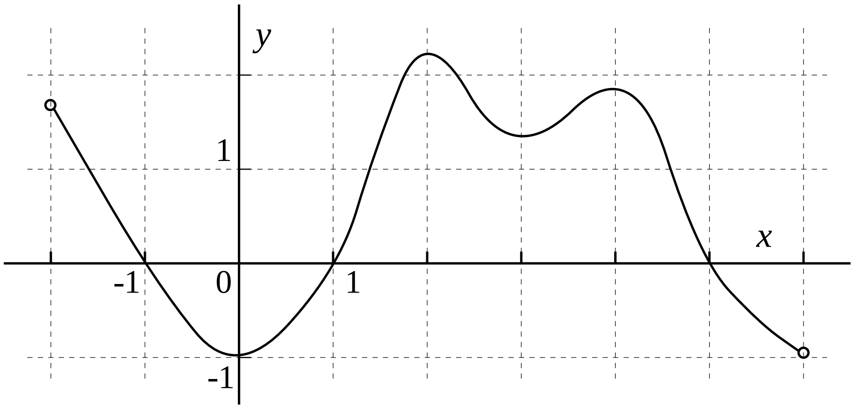
c) If $h(x) = \sqrt{3 - \sin x}$, what is $h'(x)$?

(8) 6. k is a differentiable function and the following is known about k and its derivatives.

$$k(1) = 2 \quad k'(1) = -3 \quad k''(1) = 4$$

Suppose that $j(x) = k\big(e^x\big)$ (this is a composition). Compute $j(0)$ and $j'(0)$ and $j''(0)$. Give the exact answer in each case.

(13) 7. Below is the graph of a differentiable function, Q, whose domain is $-2 < x < 6$. Use this graph to answer the following questions as accurately as you can.



The graph of $y = Q(x)$

a) For which x's is $Q(x) = 0$?

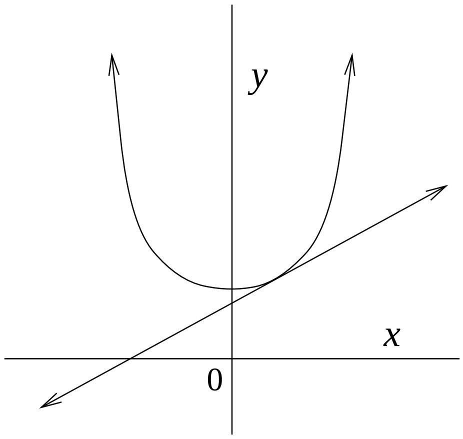
b) What are the largest intervals on which $Q(x) > 0$ for all x's in each interval?

c) For which x's is $Q'(x) = 0$?

d) What are the largest intervals on which $Q'(x) > 0$ for all x's in each interval?

(12) 8. The line $y = \frac{1}{2}x + 3$ is tangent to the parabola $y = x^2 + B$. Use algebra and calculus to find the exact value of B.

Hint: the two formulas are equal at the tangent point, and the line is tangent to the curve at that point.



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Exam 1 for Math 135

Sections 1, 2, and 4

October 12, 1998

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: ________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam or symbolic manipulation capability.

You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	20	
2	14	
3	12	
4	12	
5	9	
6	8	
7	13	
8	12	
Total Points Earned:		

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