1 B

(12) 1. Locate the exact maximum and minimum values of the function $f(x) = 3x^5 - 5x^3$ on the closed interval $[-.5, 1.5]$ using calculus. Briefly explain using calculus why the values you give are the extreme values of the function on the closed interval.

(12) 2. Suppose a point is moving along the graph of $y = x \sin x$ so that $\dfrac{dx}{dt}$ is 2 centimeters per second.

a) Find $\dfrac{dy}{dt}$ when $x = \pi$.

b) If $Q = x^2 + y^2$, is Q increasing or decreasing when $x = \pi$?

(16) 3. A dairy farmer plans to fence in a rectangular pasture adjacent to a straight river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

Explain briefly why you have found the *least* amount of fencing.

(20) 4. Suppose that $F(x) = \dfrac{x^2 - 3}{1 - x^2}$.

a) What is the domain of F? Where is $F(x) = 0$? Find any vertical asymptotes of the graph of F. Find any horizontal asymptotes of the graph of F.

b) Compute $F'(x)$ carefully. Where is $F'(x) = 0$? Analyze the behavior of F near each critical number you have found: declare whether it is a relative minimum, relative maximum, or neither, and briefly explain your reasoning.

c) Compute $F''(x)$ carefully and simplify the result. The following statement is true:

***F* has no inflection points but *F*'s concavity is different in different intervals.**

Briefly explain why each assertion in this statement is correct using your algebraic expression for $F''(x)$ and facts connecting the second derivative and concavity.

THIS PROBLEM CONTINUES ON THE NEXT PAGE WITH MORE SPACE FOR c). →

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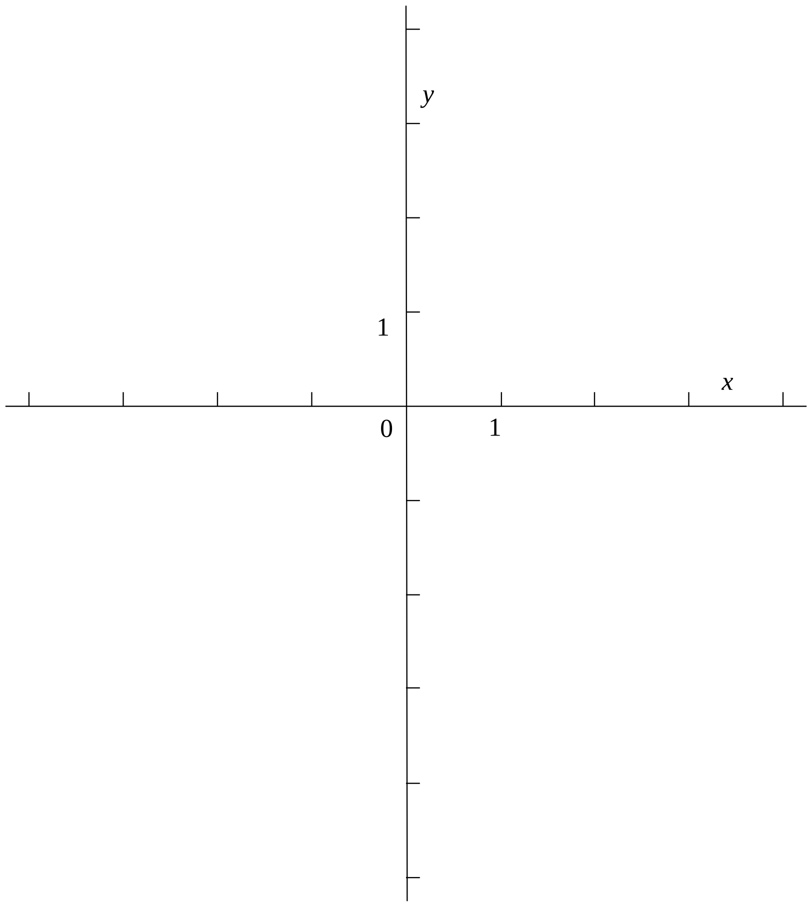
More space for the answer to c):

d) Sketch a graph of $y = F(x)$. Label on your graph each of the following:

- Intervals where F is increasing and where it is decreasing.
- Intervals where F is concave up and where it is concave down.
- Any relative maxima or minima of F.

Also include the graphs of any horizontal or vertical asymptotes.



(12) 5. Assume that the surface area in square meters S of a baby gnu is related to its weight in kilograms W by the formula

$$S = 5W^{3/2} + 3W^2 .$$

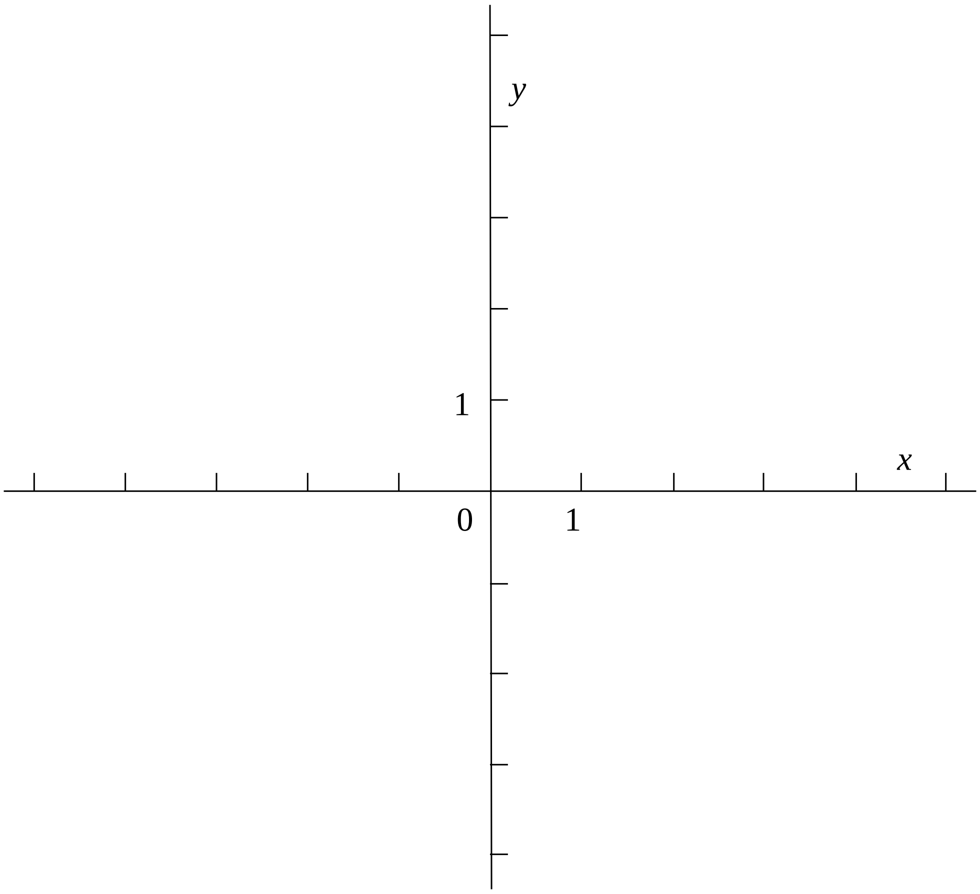
a) What is the surface area when the weight is 16 kilograms?

b) Use the differential or tangent line approximation to describe what percentage increase in surface area will result from a 5% increase in weight. Briefly explain with a graph or with further computation using calculus why your answer is greater than or less than the exact answer, which is about 9.4685%.

(14) 6. The domain of a differentiable function H is all real numbers except 0. H has the following properties.

- $\lim_{x\to 0^-} H(x) = +\infty$ and $\lim_{x\to 0^+} H(x) = -\infty$.
- $\lim_{x\to -\infty} H(x) = 2$ and $\lim_{x\to +\infty} H(x) = 1$.
- $H(x) = 0$ only when $x = -3$ and $x = -1$ and $x = 1$.
- $H'(x) = 0$ only when $x = -2$ and $x = 2$, and $H(-2) = -1$ and $H(2) = 3$.
- $H''(x) = 0$ only when $x = -4$ and $x = 3$, and $H(-4) = 1.5$ and $H(3) = 1.5$.

Sketch a graph of H. Label any relative maxima on the graph with **M** and any relative minima with **m** and any inflection points with **I**. Write the equations of any horizontal asymptotes and any vertical asymptotes.



Be sure to label any relative maxima on the graph with **M** and any relative minima with **m** and any inflection points with **I**.

Equations of any horizontal asymptotes:

ANSWER: ______________________________

Equations of any vertical asymptotes:

ANSWER: ______________________________

(14) 7. The equation

$$x + Dy^2 = e^{xy} + C$$

defines a curve for certain values of the constants C and D.

a) Find $\dfrac{dy}{dx}$ in terms of x and y and C and D (remember in this computation that C and D are unspecified constants!).

b) Find values of C and D so that the line $y = 3x + 2$ is tangent to the curve at the point $(0, 2)$.

B B

Exam 2 for Math 135

Sections 1, 2 and 4

November 23, 1998

NAME (*please print*): ____________________

SIGNATURE: ____________________

SECTION #: ____________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	12	
2	12	
3	16	
4	20	
5	12	
6	14	
7	14	
Total Points Earned:		

B B