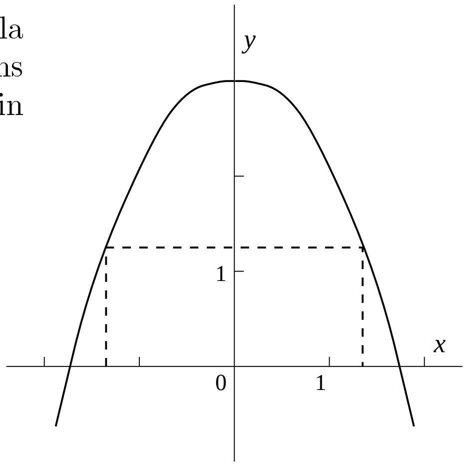
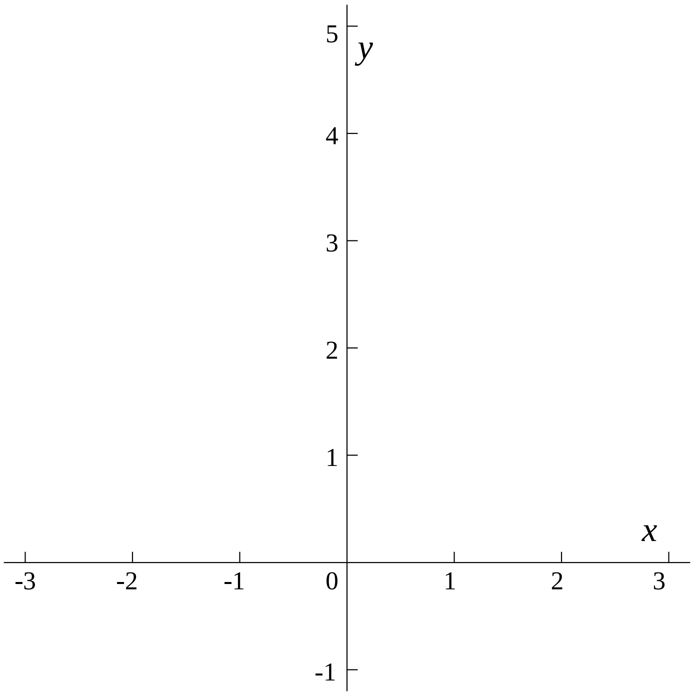
(15) 1. A rectangle is inscribed as shown below in the parabola $y = 3 - x^2$. Of all such rectangles, find the dimensions of the one whose area is a maximum. Explain briefly (in terms of calculus) why your answer gives a maximum.



(12) 2. In this problem $W(x) = \begin{cases} Ax^2 - 1 & \text{if } x < -1 \\ x + B & \text{if } -1 \le x \le 1 \\ 2 & \text{if } 1 < x \end{cases}$ where A and B are constants to be determined in part a).

a) Find numbers A and B so that the function is continuous for all values of x.

b) Sketch a graph of $y = W(x)$ using the values of A and B found in a) on the axes given.



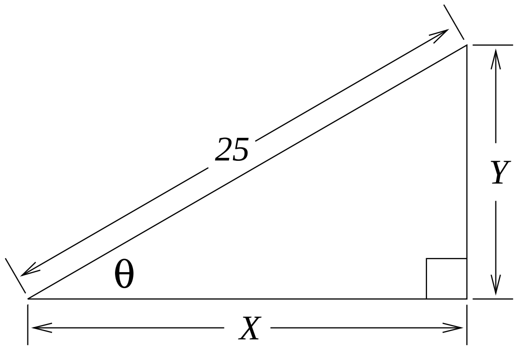
(8) 3. Evaluate the indicated limits exactly. Give brief evidence supporting your answers which is not based on a calculator graph or calculator computations.

a) $\displaystyle\lim_{x\to 5} \frac{x^2 - 3x - 10}{x - 5}$. b) $\displaystyle\lim_{x\to +\infty} \frac{2x^3 - x + 3}{x^3 + 2}$.

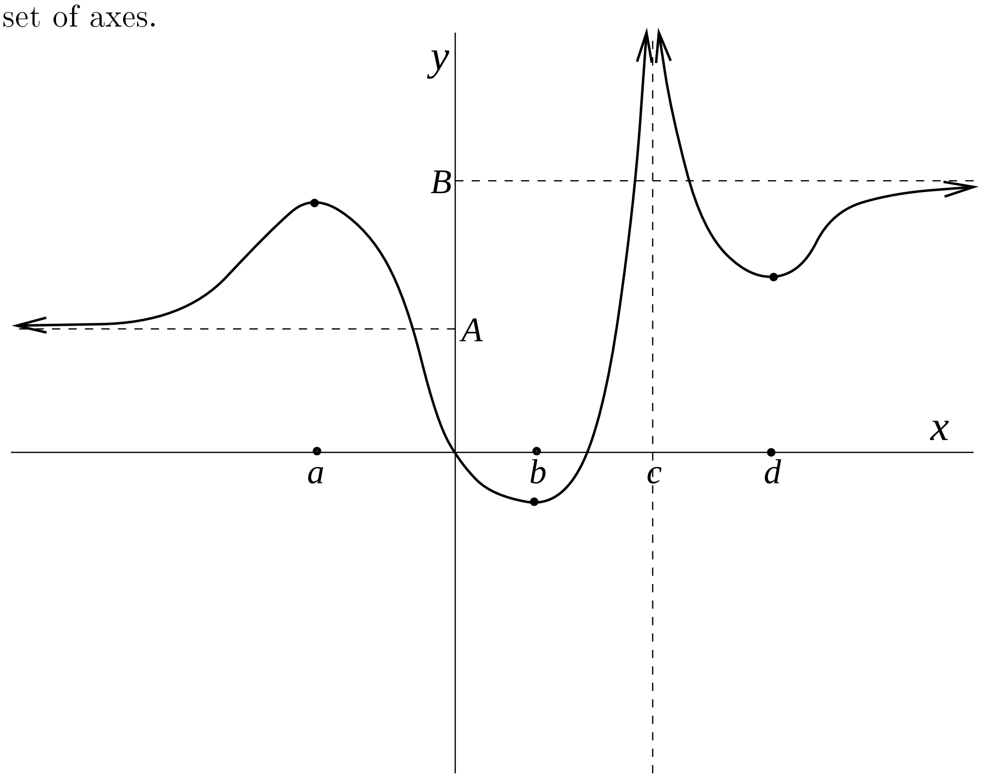
(12) 4. Find the equations of all vertical and horizontal asymptotes of the function

$$f(x) = \frac{\sqrt{x^2+4}}{x-3}.$$

(16) 5. A ladder which is 25 feet long leans against a vertical wall. The bottom of the ladder is on flat ground. Suppose that X is the distance from the bottom of the ladder to the base of the wall, Y is the distance from the top of the ladder to the base of the wall, and θ is the angle at the bottom of the ladder between the ground and the ladder. X and Y are varying with time. When $X = 15$ feet, X is increasing at 2 feet/second. What are Y and the rate of change of Y at this time? What are θ and the rate of change of θ at this time?



(12) 6. The graph of a function f is shown below. Assuming that f is differentiable everywhere except at $x = c$ and that f has the asymptotes shown, sketch a plausible graph for f' on the same set of axes.



(10) 7. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = 4 + 3x^2$.

(14) 8. Find $\dfrac{dy}{dx}$.

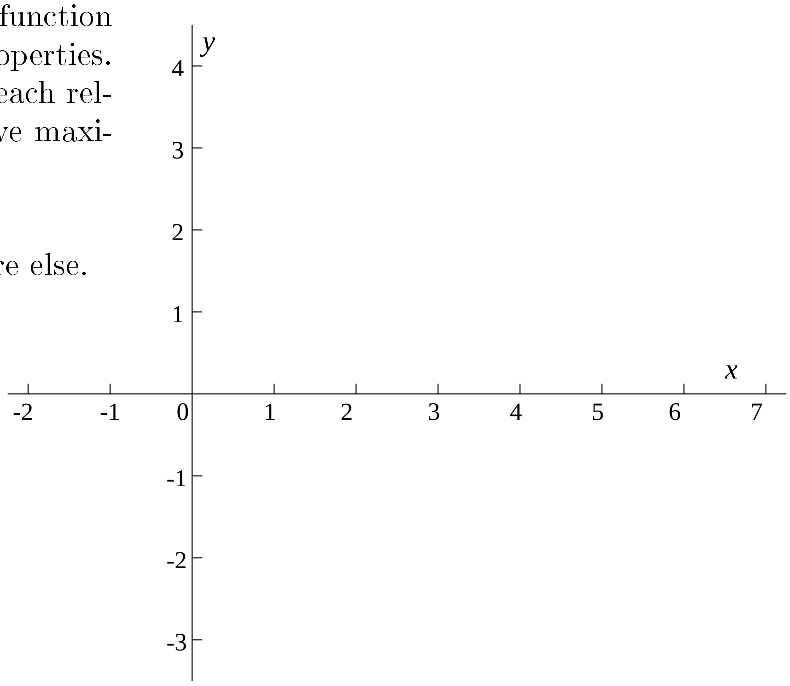
a) $y = \dfrac{3x+5}{7x^2+1}$ b) $y = x\ln(3x+5)$ c) $xe^y = \cos(xy)$

(13) 9. a) An equation of the tangent line to the graph of $y = f(x)$ when $x = 1$ is known to be $2x + y + 3 = 0$. Find $f(1)$ and $f'(1)$.

b) Suppose that $h(x)$ is a differentiable function satisfying $h'(1) = 3$ and $h'\left(\frac{\pi}{4}\right) = -2$. If $g(x) = h(\tan x)$, find $g'\left(\frac{\pi}{4}\right)$.
Computations with tangent should be simplified as much as possible. Numerical approximations are **not** acceptable.

(14) 10. Sketch the graph of a differentiable function $f(x)$ which has *all* of the following properties. Label each inflection point of f by **I**, each relative minimum by **m**, and each relative maximum by **M**.

- $f(1) = 0$.
- $f'(x) = 0$ if $x = 1$, 3, or 5, and nowhere else.
- $f'(x) > 0$ on $(1, 3)$ and $(3, 5)$.
- $f'(x) < 0$ on $(-\infty, 1)$ and $(5, \infty)$.
- $f''(x) > 0$ on $(-\infty, 2)$ and $(3, 4)$.
- $f''(x) < 0$ on $(2, 3)$ and $(4, \infty)$.



(8) 11. Find the largest and smallest values of $f(x) = 30x^3 - 90x^2 + 90x + 100$ when x satisfies $2 \leq x \leq 3$. Justify your answer by using calculus rather than a calculator graph.

(12) 12. a) If $G(x) = (2x - 1)e^{2x}$, compute $G'(x)$.

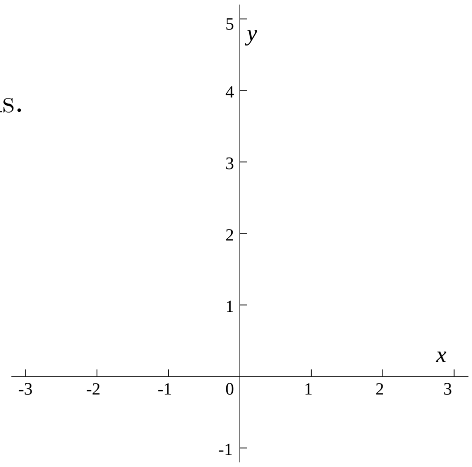
b) Use your answer to a) to evaluate $\int_0^1 xe^{2x}\,dx$ exactly.

Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(8) 13. Suppose $k''(x) = \cos x + \sin(2x)$ and $k(0) = 0$ and $k'(\pi) = 0$. Find a formula for $k(x)$. Computations with sine and cosine should be simplified as much as possible. Numerical approximations are **not** acceptable.

(20) 14. a) Sketch the region in the plane bounded above by $y = 4 - x^2$ and below by $y = 3$.

b) Find the area of this region using methods of calculus.



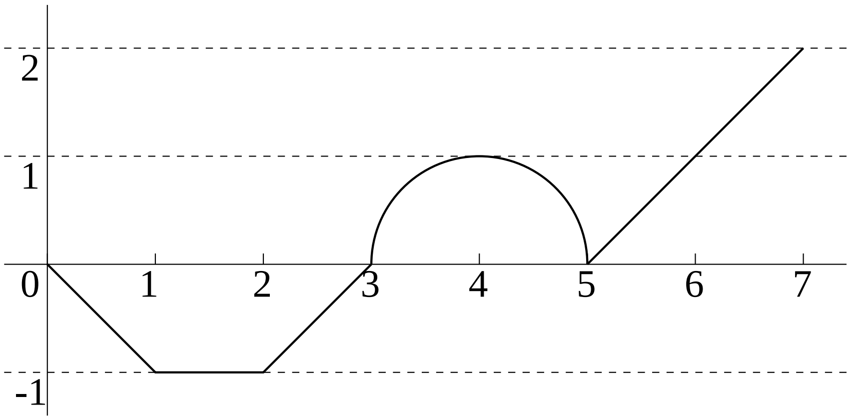
(16) 15. Evaluate the indicated integrals using methods of calculus.

a) $\displaystyle\int_1^4 \frac{1-\sqrt{x}}{x}\,dx.$

Computations with exp and log and square root should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

b) $\displaystyle\int (3x-6)e^{3x^2-12x}\,dx$

(10) 16. Below is a graph of the function R which is defined on the interval $[0, 7]$. The graph is made up of straight line segments and a semicircle.



The graph of R

a) Compute the Riemann sum estimate for $\displaystyle\int_0^7 R(t)\,dt$ associated with the partition $0 < 2 < 3 < 5 < 7$ where the sample points are the midpoints of the subintervals.

b) Compute $\displaystyle\int_1^5 R(t)\,dt.$

c) If Q is the function defined by $Q(x) = \displaystyle\int_0^x R(t)\,dt$, what is $Q'(4)$?

A **A**

Math 135 Final Exam

Wednesday, December 16, 1998, from 4 PM to 7 PM

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: __________ LECTURER: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	15	
2	12	
3	8	
4	12	
5	16	
6	12	
7	10	
8	14	
9	13	
10	14	
11	8	
12	12	
13	8	
14	20	
15	16	
16	10	
Total Points Earned:		

A **A**