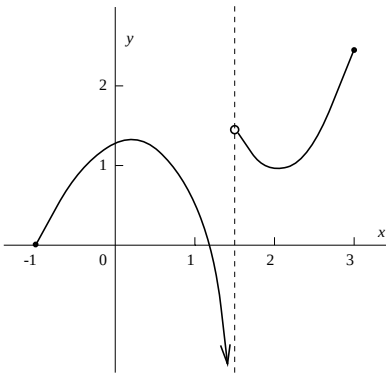
10/27/98

Review Problems for the second exam in Math 135

This collection of 16 problems is definitely longer than any "real" exam. The problems are intended to be representative of the type of problems which may appear on such exams. Many routine review problems are available in the review sections of the text (for example, the *Review Exercises for Chapter 3* appear on pages 256 through 258). Graphing calculators may be used on exams but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

1. What is the rectangle with largest area that has sides parallel to the coordinate axes and that can be enclosed between the x-axis and the curve $y = 4 - x^2$?

2. Here is a graph of $y = h(x)$, a function whose domain is $-1 \leq x < 1.5$ and $1.5 < x \leq 3$. Use this graph to sketch the graph of the derivative of h, $h'(x)$, as well as you can. Does h have any critical numbers? Does h have any relative maxima or minima? Does h have any inflection points?



y=h(x)

3. Two non-negative numbers have sum equal to a positive number, P. How should the numbers be selected so that the product of the cube of one times the square of the other is largest?

4. Find $\lim_{x\to\infty}$ and $\lim_{x\to-\infty}$ for the following functions:

a) $\dfrac{e^{2x}}{3+e^{4x}}$ b) $\dfrac{e^{4x}}{3+e^{2x}}$ c) $\dfrac{x^3+1}{x^2+x+1}$ d) $\dfrac{x+1}{\sqrt{x^2+2x+3}}$

5. At a certain time, the length of a rectangle is 3 feet and its width is 5 feet. At that time, the length is *increasing* at .4 feet per second, and the width is *decreasing* at .5 feet per second.

a) At that time, how fast is the **area** of the rectangle changing? Is the area increasing or decreasing?

b) At that time, how fast is the **diagonal** of the rectangle changing? Is the diagonal increasing or decreasing?

6. a) Graph $f(x) = e^{x-x^2}$ in the window $-2 \leq x \leq 2$ and $0 \leq y \leq 2$.

b) Calculate $f'(x)$. Find the roots of $f'(x) = 0$ algebraically. Where is $f'(x)$ positive and where is it negative? Explain how your results apply to the graph of $y = f(x)$.

c) Calculate $f''(x)$. Find the roots of $f''(x) = 0$ algebraically. Where is $f''(x)$ positive and where is it negative? Explain how your results apply to the graph of $y = f(x)$.

7. Sketch a graph of a function $W(x)$ which has all of the following properties:

- The domain of W is all numbers except 1 and W is differentiable everywhere in its domain.
- The only root of $W(x) = 0$ is at 0.
- The only roots of $W'(x) = 0$ are at -1 and 2, and $W(-1) = 2$ and $W(2) = 1$.
- The only root of $W''(x) = 0$ is at -2 and $W(-2) = 1$.
- $\lim_{x\to-\infty} W(x) = 0$, $\lim_{x\to1^-} W(x) = -\infty$, $\lim_{x\to1^+} W(x) = \infty$, and $\lim_{x\to\infty} W(x) = \infty$.

Indicate carefully any relative extrema, any inflection points, and any horizontal or vertical asymptotes.

8. A circular puddle on a flat concrete floor is spreading so that at a certain time, the radius is increasing at .3 inches per second. If the area of the puddle at that time is 25 square inches, how fast is the area changing?

9. a) Find the maximum and minimum values of the function $Q(x) = x^3 + (x-4)^2$ on the interval $[-4, 4]$.

b) Find the maximum and minimum values of the function $Q(x) = x^3 + (x-4)^2$ on the interval $[0, 4]$.

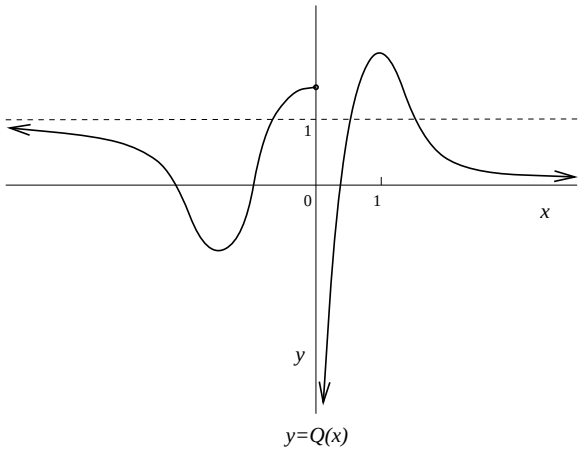
10. a) Where is $\dfrac{\sin x}{1+x^2}$ equal to 0?

b) Why am I *sure* that $\dfrac{(1+x^2)\cos x - 2x \sin x}{(1+x^2)^2}$ has at least 5 different roots inside the interval $0 < x < 20$?

Hint 20 divided by π is more than 6. This fact and a) allow you to verify b) *without* substantial work if you can see a simple relationship between a)'s function and b)'s function.

11. An open-topped box has sides costing \$10 per square foot and a square bottom costing \$20 per square foot. What is the cheapest cost for such a box whose volume is 10 cubic feet?

12. Here is a graph of $y = Q(x)$ which you should use to answer the following questions <u>as well as you can</u>. Some of your answers may have to be approximate. Note that the domain of Q is *all numbers*.



Where is Q continuous? Where is Q differentiable? What are Q's critical numbers? What are the equations of the vertical asymptotes of Q, if any? What are the equations of the horizontal asymptotes of Q, if any? What are the open intervals in which Q is {in/de}creasing? Locate and describe any relative max/min of Q. In what intervals is Q concave {up/down}? Locate any points of inflection of Q. Find these limits, if they exist: $\lim_{x\to-\infty} Q(x)$, $\lim_{x\to 0^-} Q(x)$, $\lim_{x\to 0^+} Q(x)$, and $\lim_{x\to+\infty} Q(x)$.

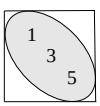
13. Consider the curve defined by the equation $5x^2 + 2xy + 2y^2 = 7$. Find $\dfrac{dy}{dx}$.

a) Which points on the curve have horizontal tangent lines?

b) Which points on the curve have vertical tangent lines?

c) The graph of this equation is a tilted ellipse. What are the corners of the smallest box with sides parallel to the coordinate axes which contains this ellipse?

Hint



14. Suppose $F(x) = \dfrac{x^3}{x^2 - x - 2}$. What is the domain of F? What are $\lim_{x\to\infty} F(x)$ and $\lim_{x\to-\infty} F(x)$? If a is a number *not* in F's domain, what are $\lim_{x\to a^-} F(x)$ and $\lim_{x\to a^+} F(x)$? If there is more than one number not in F's domain, compute these limits for each number not in the domain of F. Does F have any vertical or horizontal asymptotes, and, if it does, what are they? Find F' and simplify it. Where is $F'(x) = 0$? If F has any relative extrema, what and where are they? In what intervals is F {in/de}creasing? Find F'' and simplify it. Where is $F''(x) = 0$? Find any inflection points of F. In what intervals is F concave {up/down}? Now <u>use all of this information</u> to sketch a reasonable graph of F.

15. Find the range of $V(x) = e^{-2x} + e^{3x}$.

Note Your answer should *not* be approximate, but should be written in terms of values of well-known functions. Expressions involving ln and exp should be simplified as much as possible.

16. Suppose that W is a differentiable function with $W(3) = 4$, $W'(3) = 2$ and $W''(3) = -5$. Use linear approximation or differentials to find approximate values for $W(3.03)$ and $W(2.98)$. Use the information given to decide if these approximate values are likely to be greater than or less than the true values of W. Briefly explain your answer.