Comments on the Syllabus

Precalculus review, lectures 1–3

We *must* assume that students in Math 135 have an adequate background in plane and analytic geometry and algebra, including manipulation of exponentials and logarithms. They *must* also have some knowledge of trigonometry. They *must* know what a function is, what its graph is, and the appropriate notation, along with a comprehensive collection of examples. The portion of the Math 135 syllabus listing a review of precalculus topics is primarily a gesture of good will, intended to insure that everyone involved with the course uses the same notation and the same vocabulary. These lectures are *not* meant to substitute for previous study and good understanding of the prerequisites! If this material is *not* familiar to a student, then that student should reconsider participation in the course. The course moves very fast and there isn't time to learn the prerequisites.

Extended discussion of the $\varepsilon-\delta$ formulation of limits, lecture 5

Limits weren't understood well centuries ago when calculus began. They were mocked as the "ghosts of departed quantities". Precise definition of the limit concept may increase serious students' appreciation of calculus. Limits in this course will principally be a language to deal with asymptotic behavior. This language will be supported by algebraic manipulation ($\lim\limits_{x\to 3}\frac{x^2-x+6}{x-3}=\lim\limits_{x\to 3}\frac{(x-3)(x+2)}{x-3}=\lim\limits_{x\to 3}x+2=5$) and by numerical or graphical analysis ($\lim\limits_{h\to 0}\frac{e^h-1}{h}=1$). Much class time or homework dealing with the precise definition of limit and its verification is probably not useful.

Limits at infinity can be infinity, lecture 17

Section 3.5 of the text discusses such limits as $\lim\limits_{x\to+\infty}\frac{5x+12}{\sqrt{7x^2+3x+20}}$. This limit is $\frac{5}{\sqrt{7}}$ and the limit statement can be interpreted geometrically by noting that $y=\frac{5}{\sqrt{7}}$ is a horizontal asymptote of the corresponding graph. Again, limits will be used in this course as a convenient language for asymptotic behavior. What happens to x^5 when x is positive and large? When x has that sign and size behavior, then x^5 must also be large and positive. This may be conveniently abbreviated by $\lim\limits_{x\to+\infty}x^5=+\infty$. This equation is only a slight extension of such statements as the text's $\lim\limits_{x\to 0^+}\frac{1}{x}=+\infty$. If we allow $+\infty$ and $-\infty$ to be values of $\lim\limits_{x\to-\infty}f(x)$ and $\lim\limits_{x\to+\infty}f(x)$ then we can describe briefly whether graphs ultimately trend up or trend down, far to the left or far to the right. We still do not allow $\lim\limits_{x\to+\infty}\sin x$ to have a value – so this last limit "Does not exist". The textbook asserts that $\lim\limits_{x\to+\infty}x^5$ also "Does not exist" but the change suggested here may be useful. Note that the only change in homework answers seems to be that the answer to problem 13 of section 3.5 becomes $-\infty$ rather than "Does not exist".

Exponential growth is faster than any polynomial growth, lectures 17 & 18

The exponential function grows more rapidly than any polynomial. No section of the textbook used in Math 135 mentions this, but knowing facts like $\lim\limits_{x\to\infty}\frac{x^{100}}{e^x}=0$ can be very helpful in applications. Instead of x^{100}, *any* power of x can be used, and instead of e^x *any* increasing exponential (e^{ax} with $a > 0$) can be used. The limit given can be analyzed by writing $\frac{x^{100}}{e^x}$ as the product $\frac{x^{100}}{e^{.5x}} \cdot \frac{1}{e^{.5x}}$ and then recognizing that the first factor is positive and decreasing after its positive critical number and that the second factor $\to 0$. These limits discussed here are also relevant to problems 23 and 25 of section 3.6 which are not part of the official syllabus. Some students try them anyway and similar problems are sometimes given on exams.

Business applications

Section 3.10 discusses various elementary applications of calculus to business and economics. This may be interesting to some Math 135 students who have heard of marginal profit, marginal revenue, etc. This section can't be covered formally, but students should be aware of it.

Understanding the error in linear approximation, lecture 21

Estimation of $f(x + \triangle x)$ by $f(x) + f'(x)\triangle x$ is called "the differential" in the text. This estimation is also called "linear approximation" or "the tangent line approximation". The Mean Value Theorem can be applied to analyze the error in this estimation. The error is directly proportional to the product of f''(a number between x and $x + \triangle x$) (with the sign of this value of f'' indicating whether the estimate is an over- or underestimate) and of $(\triangle x)^2$, so that for small $\triangle x$'s the error is very small. A sketch of the curve and the way it bends (its concavity) at x shows that whether $f(x) + f'(x)\triangle x$ is an over- or underestimate of $f(x + \triangle x)$ when $\triangle x$ is small depends on the sign of $f''(x)$.

Vocabulary: "sample points" for Riemann sums, lecture 25

A Riemann sum for a function f on an interval $[a, b]$ needs a great deal of specification. The interval $[a, b]$ must be divided by a *partition* $\{a = x_0, x_1, \ldots, x_n = b\}$. Additionally, the function must be evaluated at chosen points in each subinterval created by the partition. For example, the number c_1 is selected to be in the subinterval $[x_0, x_1]$ and $f(c_1)$ is computed and multiplied by $x_1 - x_0$, the number c_2 is selected to be in the subinterval $[x_1, x_2]$ and $f(c_2)$ is computed ... The text does not give a name to the collection of the points, $\{c_1, c_2, \ldots, c_n\}$. In this course the numbers $c_1, c_2, \ldots, c_n$ where each c_i is restricted to be in the appropriate subinterval determined by the partition will be called the *sample points* of the Riemann sum.

Using Σ notation for Riemann sums, lecture 25

The use of Σ notation for Riemann sums may be a novelty to many students in the course. It is a novelty whose secrets aren't worth learning here. But learning about some natural ways that Riemann sums arise, such as the present or future value of an income stream or the current population of a species as a result of summing birth/death rates or net fluid flow through an artery, is a better use of class time for most students than discussing the text's Theorem 4.2 which gives formulas for some sums of powers. Students should know that Riemann sums can analyze many phenomena in addition to approximating area.