1 A

(20) 1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\displaystyle\lim_{x\to 2} \frac{x^2+x-6}{x-2}$

b) $\displaystyle\lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

c) $\displaystyle\lim_{x\to \frac{\pi}{2}^-} \frac{\sin x}{x-\frac{\pi}{2}}$

d) $\displaystyle\lim_{x\to 0^+} \frac{|x|}{x}$

(14) 2. a) Suppose $f(x)$ is a function. Write the **definition** of $f'(x)$, the derivative of $f(x)$, as a limit.

b) Use your answer to a) and familiar properties of limits to find the derivative of $f(x) = \sqrt{x-3}$.

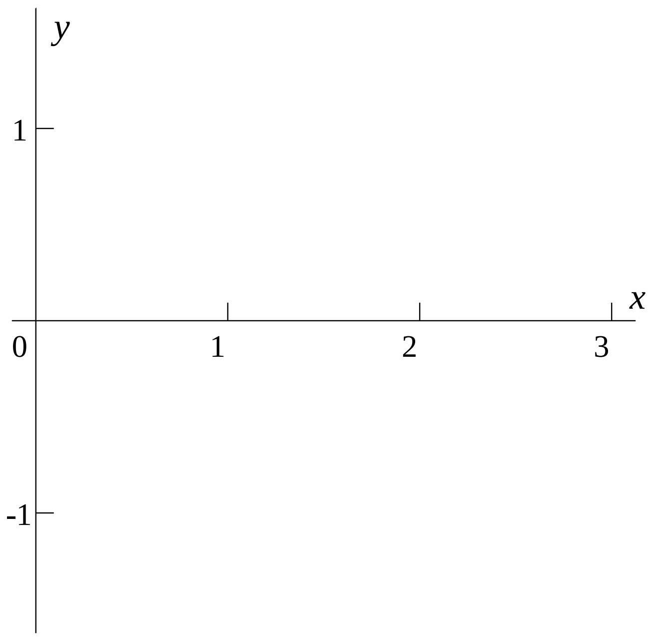
c) Use your answer to b) to write an equation for the line tangent to $y = \sqrt{x-3}$ when $x = 7$.

(14) 3. Suppose that the function G is described by

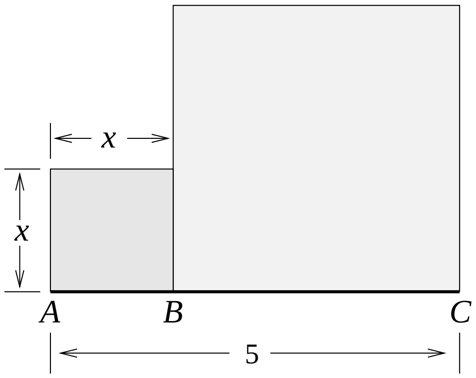
$$G(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ Ax^2 + B & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } 2 < x \leq 3 \end{cases}.$$

a) Find A and B so that G is continuous for all numbers in its domain. Briefly explain your answer.

b) Graph $y = G(x)$ on the axes given for the values of A and B found in a).

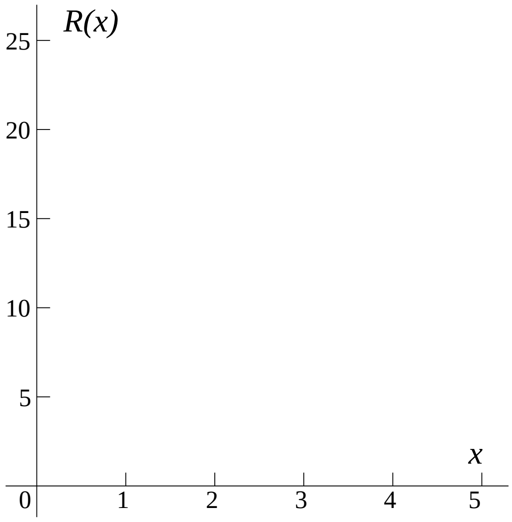


(16) 4. The line segment $\overline{AC}$ is 5 units long and is divided into two parts by the point B. Suppose that x is the length of the segment $\overline{AB}$ and that $R(x)$ is the sum of the areas of the two squares with sides $\overline{AB}$ and $\overline{BC}$ as shown.



a) Write an algebraic formula for $R(x)$ as a function of x.

b) Graph $R(x)$ on the axes shown. Note that the horizontal and vertical axes have different scales.



c) Use algebra to find all exact values of the length x for which $R(x) = 17$.

(9) 5. Use all differentiation rules here. Please do *not* simplify the answers in this problem!

a) If $P(x) = 6x^5 - 3x^3 + 4$, what is $P'(x)$?

b) If $Q(x) = \dfrac{x^3+1}{3x-2}$, what is $Q'(x)$?

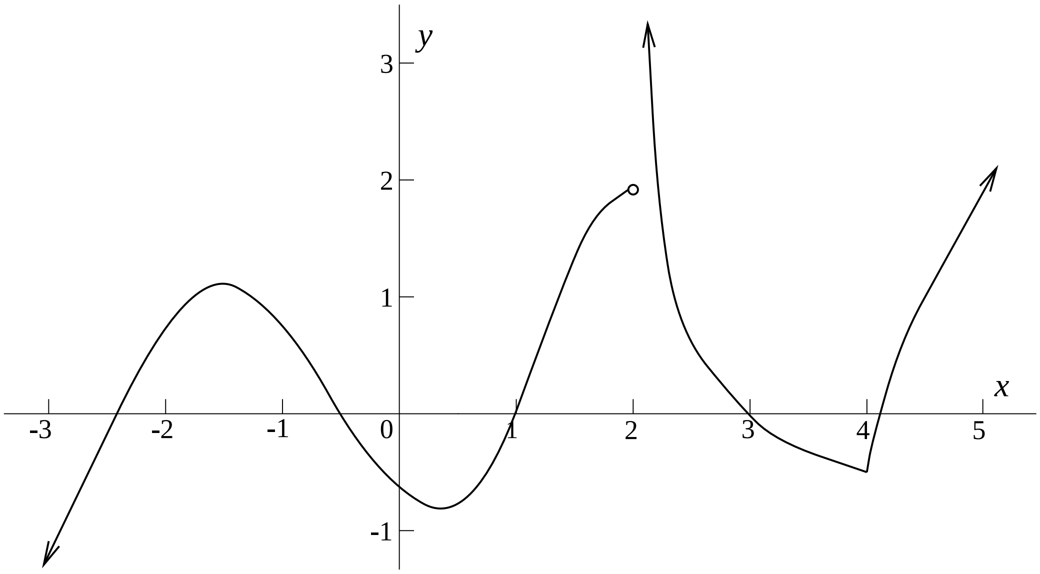
c) If $R(x) = (5 + \cos x)^{100}$, what is $R'(x)$?

(8) 6. Suppose that T is a differentiable function and the following facts are known about T and its derivatives:

$$T(5) = 0 \text{ and } T'(5) = -3 \text{ and } T''(5) = 2\,.$$

Suppose that $S(x) = e^{T(x)}$ (this is a composition). Compute $S(5)$ and $S'(5)$ and $S''(5)$. Give the exact answer in each case.

(19) 7. Below is a graph of the function F. The domain of F is <u>all numbers not equal to 2</u>.



The graph of $y = F(x)$

Use this graph to answer the questions below as accurately as possible.

a) For which x is $F(x) = 0$?

ANSWER ______________________________

b) For which x is $F(x) > 0$?

ANSWER ______________________________

c) What is $\lim_{x \to 2^-} F(x)$?

ANSWER ______________________________

d) What is $\lim_{x \to 2^+} F(x)$?

ANSWER ______________________________

e) For which x (in its domain) is F <u>not</u> differentiable?

ANSWER ______________________________

f) For which x is $F'(x) = 0$?

ANSWER ______________________________

g) What is $F(x)$ for those x's for which $F'(x) = 0$?

ANSWER ______________________________

h) For which x is $F'(x) > 0$?

ANSWER ______________________________

A A

Exam 1 for Math 135

Sections 8, 9, and 10

February 23, 1999

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: ______________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam or symbolic manipulation capability.

You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	20	
2	14	
3	14	
4	16	
5	9	
6	8	
7	19	
Total Points Earned:		

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