Function Derivative

Function	Derivative
C	0
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$1/x$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$(\sec x)^2$

If $\lim_{x\to a^{(\pm)}} f(x) = \pm\infty$ then $x = a$ is a **vertical asymptote** of $y = f(x)$ and if $\lim_{x\to\pm\infty} f(x) = b$ then $y = b$ is a **horizontal asymptote** of $y = f(x)$.

Function	**Derivative**
$Kf(x)$	$Kf'(x)$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)\cdot g(x)$	$f'(x)\cdot g(x)+f(x)\cdot g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x)-g'(x)f(x)}{g(x)^2}$
$f(g(x))$	$f'(g(x))\cdot g'(x)$

Function	**Domain**	**Range**	**Graph**
x^2	all x	$y \ge 0$	
x^3	all x	all y	
$\sqrt{x}$	$x \ge 0$	$y \ge 0$	
$\lvert x\rvert$	all x	$y \ge 0$	
$1/x$	$x \ne 0$	$y \ne 0$	
$\sin x$	all x	$-1 \le y \le 1$	
$\cos x$	all x	$-1 \le y \le 1$	
$\tan x$	$x \ne (\text{odd int})\frac{\pi}{2}$	all y	
$\ln x$	$x > 0$	all y	
e^x	all x	$y > 0$	

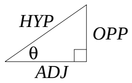
Logarithmic properties

$\ln(a\cdot b) = \ln a + \ln b$ $\ln(a^b) = b\ln(a)$
$\ln(a/b) = \ln(a) - \ln(b)$ $\ln(\frac{1}{b}) = -\ln(b)$
$\ln(e^a) = a$ $\ln(1) = 0$ $\ln(e) = 1$

Exponential properties

$a^{b+c} = a^b\cdot a^c$ $a^{-b} = 1/a^b$
$(a^b)^c = a^{bc}$ $e \approx 2.718$
$a^0 = 1$ $e^{\ln a} = a$ if $a > 0$

Triangle things 360^o (degrees) $= 2\pi$ radians



$\sin\theta = \frac{OPP}{HYP}$
$\cos\theta = \frac{ADJ}{HYP}$
$\tan\theta = \frac{OPP}{ADJ}$

Pythagoras

$(ADJ)^2+(OPP)^2=(HYP)^2$
$(\sin\theta)^2 + (\cos\theta)^2 = 1$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	*NONE*
π	0	-1	0

More formulas

The **roots** of $ax^2+bx+c=0$ are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
Distance from (a,b) to (c,d): $\sqrt{(a-c)^2+(b-d)^2}$
Circle center (h,k) & radius r: $(x-h)^2+(y-k)^2 = r^2$
Line $y = mx+b$ and $m = \frac{y_2-y_1}{x_2-x_1}$ (**slope** of the line)
Addition formulas $\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$
Periodicity $\sin(x+2\pi) = \sin x$ and $\cos(x+2\pi) = \cos x$ and $\tan(x+\pi) = \tan x$ for all x

Area and Volume Formulas

Triangle $A = \frac{1}{2}$ BASE·HEIGHT
Rectangle $A =$ LENGTH·WIDTH
Circle $A = \pi$ RADIUS^2
Circle $C = 2\pi$ RADIUS
Box $V =$ LENGTH·WIDTH·HEIGHT
Cylinder $V = \pi$ RADIUS^2·HEIGHT
Sphere $A = 4\pi$ RADIUS^2
Sphere $V = \frac{4}{3}\pi$ RADIUS^3

w in f's domain is a **critical number** if **either** $f'(w) = 0$: could look like $\pm$ or $\pm$ **or** $f'(w)$ doesn't exist: might look like $\pm$ or $\pm$ or even like $\pm$ if f isn't continuous at w.

Finding max/min on a closed interval If f is continuous on $a \le x \le b$ then f's max/min values must occur either at a or at b or at a critical number inside the interval.

The First Derivative Test A critical number w is a **relative max** if $f'(\text{left of } w) > 0$ & $f'(\text{right of } w) < 0$; **relative min** if $f'(\text{left of } w) < 0$ & $f'(\text{right of } w) > 0$. **Important** No other critical numbers should be between w and where the sign of f' is checked! If both are positive or both are negative, then w is an **inflection point** of f.

The Second Derivative Test A critical number w is a **relative min** if $f''(w) > 0$ & **relative max** if $f''(w) < 0$.

f has an **inflection point** at w if w is in f's domain and if the concavity of f's graph is different on either side of w: $\pm$ (here $f''(w) = 0$) or $\pm$ (here $f''(w)$ doesn't exist).

f is **continuous** at w if $\lim_{x\to w} f(x)$ exists and equals $f(w)$ **or** check $\lim_{x\to w^+} f(x)$ and $\lim_{x\to w^-} f(x)$ both exist and $= f(w)$. f is **differentiable** at w if $\lim_{h\to 0}\frac{f(w+h)-f(w)}{h}$ exists. This is $f'(w)$: **the rate of change of f with respect to w** or **the slope of the tangent line** to $y = f(x)$ at $x = w$.

Implicit differentiation/related rates
Key point Differentiate a whole equation. Don't forget what's varying, chain rule, product rule, etc. **Example** If $xy^2 = \sin(x+y)+3x$ then $\frac{d}{dx}$ the equation. Get $1\cdot y^2+x\cdot 2yy' = \cos(x+y)(1+y')+3$. **Solve** for y'.

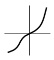
f defined in $a < x < b$ has a **relative maximum** at w in the interval if $f(w) \ge f(x)$ for x's near w on both sides. f defined in $a < x < b$ has a **relative minimum** at w in the interval if $f(w) \le f(x)$ for x's near w on both sides. Relative max and min must occur **at critical numbers**.

Differential or tangent line approximation
$f(w+\triangle w) \approx f(w) + f'(w)\triangle w$. The graph's bending causes **error**: the true value is larger when the graph is concave up and smaller when the graph is concave down.

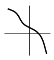
Intermediate Value Theorem If f is continuous in $a \le x \le b$, f's values include all numbers between $f(a)$ and $f(b)$: a continuous function's graph has no jumps. **Mean Value Theorem** If f is differentiable in $a \le x \le b$, there are some c's in the interval with $f'(c) = \frac{f(b)-f(a)}{b-a}$: some tangent lines of a differentiable function's graph must be parallel to any chord. **Rolle's Theorem** MVT with $f(a) = f(b) = 0$. **Fund. Thm. of Calculus** If $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$; $\frac{d}{dx}\int_a^x f = f(x)$.

Function	Antiderivative
$f(x)$	$\int f(x)\,dx$
$K f(x)$	$K \int f(x)\,dx$
$f(x) + g(x)$	$\int f(x)\,dx + \int g(x)\,dx$
x^n	$\frac{1}{n+1}x^{n+1} + C$, $n \ne -1$
$\frac{1}{x}$	$\ln x + C$ $(x > 0)$
e^x	$e^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\tan x$	$\ln(\sec x) + C$
$\int f(u)\,du$	$\int f(g(x))\,g'(x)\,dx$
when $u = g(x)$ (**Substitution**)	

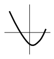
f is **increasing** in $a < x < b$ if $f(x_1) \le f(x_2)$ for any $x_1 \le x_2$ in the interval. If $f'(x) > 0$ always in $a < x < b$ then f is increasing there.



f is **decreasing** in $a < x < b$ if $f(x_1) \ge f(x_2)$ for any $x_1 \le x_2$ in the interval. If $f'(x) < 0$ always in $a < x < b$ then f is decreasing there.

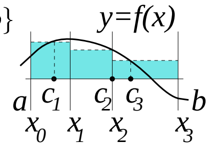


f is **concave up** if lines connecting the graph are above the graph: it bends *up*. If $f''(x) > 0$ always in $a < x < b$, f is concave up.

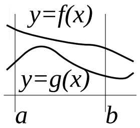


f is **concave down** if lines connecting the graph are below the graph: it bends *down*. If $f''(x) < 0$ always in $a < x < b$, f is concave down.

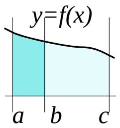
Sample points $\{c_1, c_2, c_3\}$ **Partition** $\{a = x_0 \le x_1 \le x_2 \le x_3 = b\}$
Riemann sum $f(c_1)(x_1 - x_0) + f(c_2)(x_2 - x_1) + f(c_3)(x_3 - x_2)$
With many sample points and small differences in the partition, the sum will closely approximate the **definite integral** $\int_a^b f(x)\,dx$.



If $g(x) \le f(x)$ in $a \le x \le b$, $\int_a^b g(x)\,dx \le \int_a^b f(x)\,dx$.



$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



$\int_a^b f(x)\,dx$ is **signed area**: area **I** $-$ area **II** $+$ area **III**.