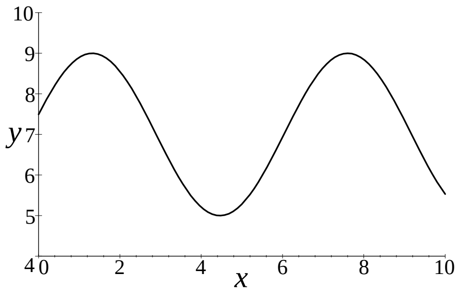
Review Problems for the first exam in Math 135 in the Spring 1999 semester

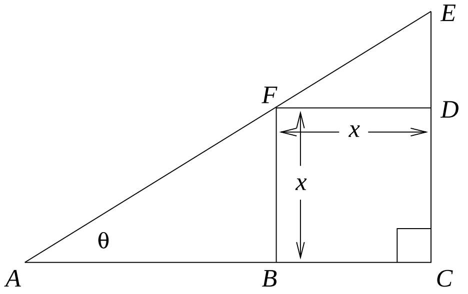
Please note that this collection of 18 problems is definitely longer than any "real" exam. The problems are intended to be representative of the type of problems which may appear on such exams.

Graphing calculators may be used on exams but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

1. Here is a graph of $y = A + B\sin(x + C)$. Estimate values of A, B, and C from the graph.

Note that the graph displayed is in the window $0 \le x \le 10$ and $4 \le y \le 10$.



2. 

Suppose that the line segment $\overline{AB}$ has length 12 and the line segment $\overline{DE}$ has length 5 in the right triangle shown. What is the side length x of the square? (Use similar triangles.) What is the length of the largest hypotenuse, $\overline{AE}$? What is the angle θ?

3. a) Simplify $e^{3\ln\sqrt{x}}$ (give an expression not involving any exponential or logarithmic function).

b) If $\ln A = 1.2$ and $\ln B = 5.3$, compute $\ln\left(B^2\sqrt{A}\right)$ exactly. Also, is A^9 greater than or less than B^2?

4. a) Find an equation of the circle C whose center is at $(2,0)$ and whose radius is 3.
b) Find an equation of the straight line L which goes through the points $(3,0)$ and $(0,6)$.
c) Find the coordinates of the points where the straight line L and the circle C intersect. Sketch the line and the circle on the same coordinate axes.

5. A function Z, defined for $x \ge 0$, is given by $Z(x) = \begin{cases} 2x & \text{if } 0 \le x \le 3 \\ x+3 & \text{if } 3 < x \end{cases}$.

a) Sketch the graph of $y = Z(x)$.
b) Compute $Z(2)$ and $Z(4)$.
c) Suppose the function inverse to Z is called N. What is $N(8)$? What is $N(2)$?
d) Sketch the graph of $y = N(x)$.

6. Suppose $p = 100 + 3\sqrt{n}$ represents the production cost, p, in dollars of n boxes of detergent when n satisfies the restriction $1 \le n \le 100$. Solve the equation for n as a function of p. What is the domain of this function of p? Describe what the formula you have obtained represents in terms of detergent and production costs. Be as specific as you can.

7. Use algebraic methods developed in this course to find the following limits, if they exist, and give evidence supporting your answers: a) $\displaystyle\lim_{x\to 2}\frac{x-2}{x^2+4}$ b) $\displaystyle\lim_{x\to 2}\frac{x-2}{x^2-4}$ c) $\displaystyle\lim_{x\to 4}\frac{x-4}{\sqrt{x}+2}$ d) $\displaystyle\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}$.

8. Write an equation of the line tangent to the graph of the function $f(x) = 3x^2 + 4$ when $x = 1$.

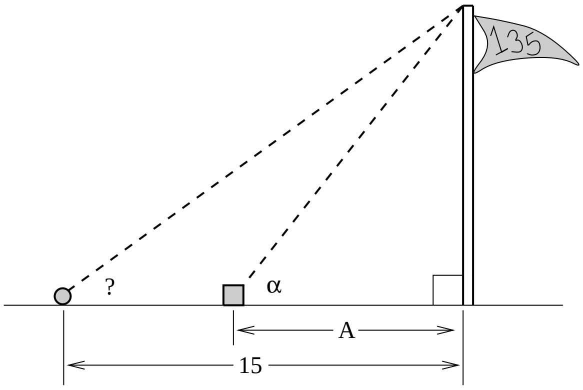
9. Use the definition of derivative and familiar properties of limits to find the derivative of $f(x) = \dfrac{3}{x}$.

10. Compute the derivative of these functions: a) $x^3 - 3x + 17$ b) $e^x \sin x$ c) $\dfrac{2x+3}{x^2+1}$ d) $3e^x + \dfrac{x}{2+5\cos x}$.

11. Suppose f and g are differentiable functions such that $f(g(x)) = 8x^2$ for all real numbers x. Assume that $f(2) = 7$, $g(2) = 4$, and $f'(4) = 2$. What is $g'(2)$?

12. A flag pole which is 10 feet tall is placed vertically on flat ground.

a) If a ball on the ground is 15 feet away from the base of the flag pole, what is the angle seen at the ball from the base of the flag pole to the top of the flag pole?
b) If a box on the ground is A feet from the base of the flag pole, write an equation for the angle α seen at the box from the base of the flag pole to the top of the flag pole in terms of A.



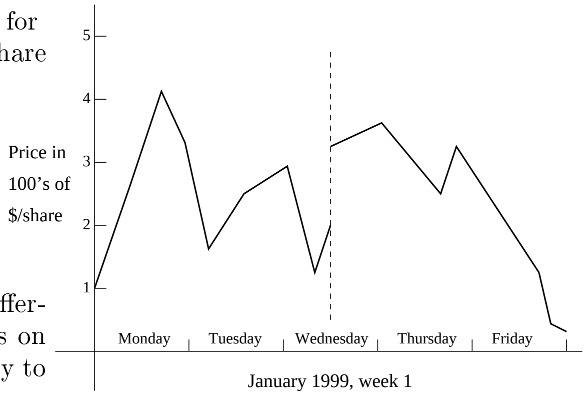
13. Let $f(x) = \begin{cases} x^2+2, & x<0 \\ x+A, & 0 \le x < 2 \\ x^2+1, & x \ge 2, \end{cases}$ where A is a number to be determined.

a) Find A such that for this particular A, $f(x)$ is continuous at $x=0$.
b) For the A you obtained in part a), is f also continuous at $x=2$? Explain your answer clearly.

14. The short-lived commercial venture 135.com* existed for only one week in January. Various events affected the share price of 135.com:

A A rumor that the course would be required by *every* major at Rutgers with a final grade of "B" or better.
B Anything with ".com" in it is a great stock.
C Only majors in Criminal Forestry would have to take Math 135.
D A company selling calculus can't be a very good investment.
E Identical final exams every semester, producing uniformly high grades with consequences for enrollment and re-enrollment.

When is the price per share continuous? When is it differentiable? Briefly explain your answers. To which points on the stock chart would each news item listed above be likely to correspond?

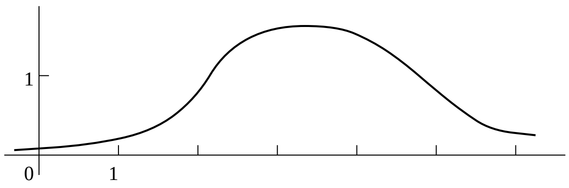


Share price of 135.com

15. Draw the graph of a function h with *all* these properties; write the equations of any vertical asymptotes.

$$\left[\text{The domain of } h \text{ is } 0<x<3 \text{ and } 3<x<5;\ h \text{ is continuous everywhere in its domain; } \lim_{x\to 0^+} h(x) = +\infty \text{ and } \lim_{x\to 3^-} h(x) = +\infty \text{ and } \lim_{x\to 3^+} h(x) = -\infty \text{ and } \lim_{x\to 5^-} h(x) = 1;\ h \text{ is differentiable everywhere in its domain, and the only } x\text{'s for which } h'(x)=0 \text{ are 2 and 4; } h(2)=1 \text{ and } h(4)=2.\right]$$

16. This graph represents the concentration of enzyme in the bloodstream as a function of time. For each of the following statements, use the graph to find a time interval in which the statement is true. Explain your assertions.
i) Enzyme concentration is increasing, but the rate of increase is slowing.
ii) Enzyme concentration is increasing and the rate of increase is also increasing.



Enzyme concentration as a function of time in seconds

17. Suppose that f and g are differentiable functions, and the following information is known:

$$f(1)=2;\quad f'(1)=-3;\quad g(1)=5;\quad g'(1)=7.$$

If $F(x)=f(x)g(x)$ and $G(x)=\dfrac{f(x)}{g(x)+x^2}$, compute $F(1)$, $F'(1)$, $G(1)$, and $G'(1)$.

18. Find $\dfrac{dy}{dx}$ when y is each of the following: a) $y=e^{3\cos 2x}$ b) $y=\ln(x^3+5x+9)$ c) $y=\sin\left(\dfrac{x+1}{x^2+2}\right)$.

On the web at `http://www.math.rutgers.edu/math135.html`

* Simulated. Your returns may vary. Please consult your own investment and tax advisors