(8) 1. Evaluate the indicated limits exactly. Give brief evidence supporting your answers which is not based on a calculator graph or calculator computations.

a) $\lim_{x\to 9} \frac{2x-18}{\sqrt{x}-3}$.

b) $\lim_{x\to +\infty} \frac{x^2+3}{x^2-2x}$.

(12) 2. Find the equations of all vertical and horizontal asymptotes of the function

$$f(x) = \frac{3+5e^{-2x}}{7-e^{-2x}} .$$

Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(10) 3. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \frac{1}{3x+4}$.

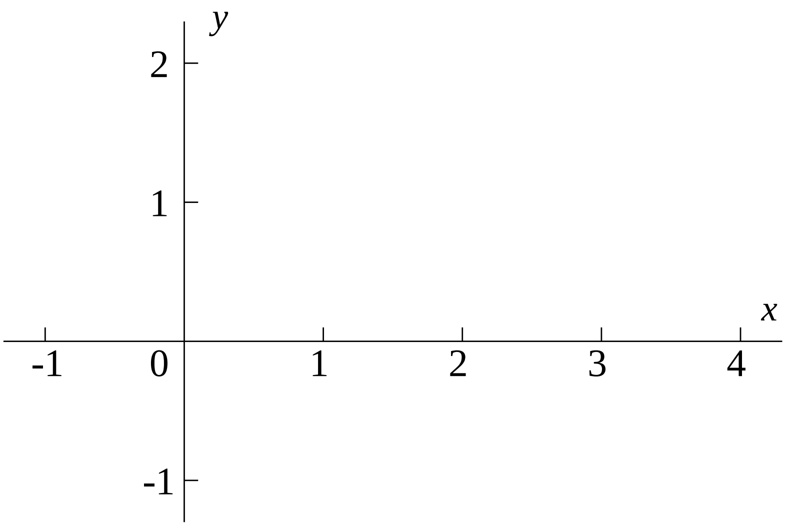
(12) 4. In this problem

$$W(x) = \begin{cases} 1 & \text{if } x \le 1 \\ \frac{A}{x}+B & \text{if } 1 < x < 2 \\ x-2 & \text{if } 2 \le x \end{cases}$$

where A and B are constants to be determined in part a).

a) Find numbers A and B so that the function is continuous for all values of x.

b) Sketch a graph of $y = W(x)$ using the values of A and B found in a) on the axes given.



(14) 5. Find $\frac{dy}{dx}$.

a) $y = \dfrac{3x^3 - 1}{3x^4 + 1}$

b) $y = \ln(x^4 + 3x + 1)$

c) $2x^3 + 5x^2 y + y^3 = 2$

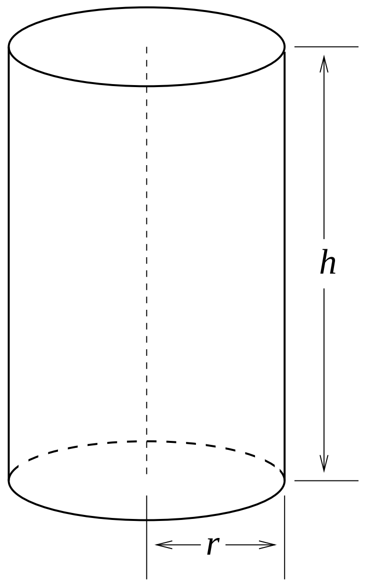
(15) 6. a) Suppose that U is a differentiable function with $U(8) = 5$ and $U'(8) = 3$, and that $V(x) = U\big(x^3\big)$. What are $V(2)$ and $V'(2)$?

b) Use the information obtained in a) to write an equation of the line tangent to the graph of the function $y = V(x)$ at the point where $x = 2$.

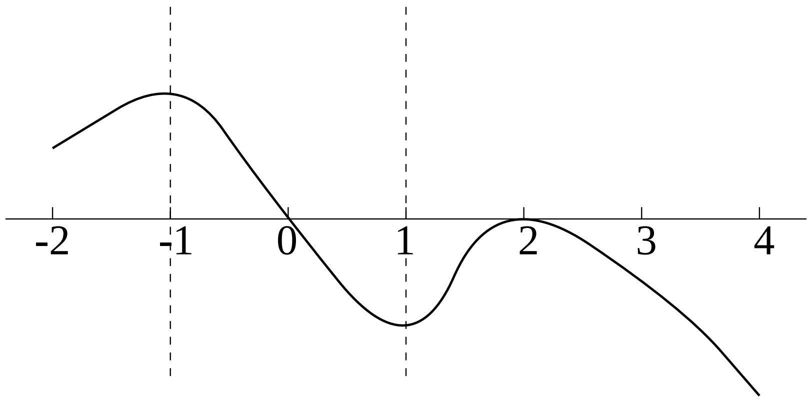
(14) 7. The volume V of a circular cylinder which has height h and base radius r is given by the equation $V = \pi r^2 h$.

A piece of clay in the shape of a circular cylinder always has *constant volume* equal to 20π cubic inches. At a certain time, the height of the cylinder is 5 inches and the cylinder is being stretched so that the height is *increasing* at the rate of .7 inches per minute.

- What is the radius of the base of the cylinder at that time?
- How fast is the radius of the base of the cylinder changing at that time?
- Is the radius of the base increasing or decreasing at that time?



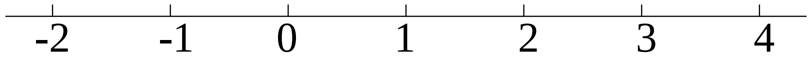
(26) 8. A certain function h is known to have continuous first and second derivatives on the interval $[-2, 4]$. The graph of the first derivative, h', is given by:



The graph of $y = h'(x)$

a) Based on the evidence given, what are the intervals on which h is increasing? Based on the evidence given, what are the intervals on which h is decreasing? Briefly explain your assertions.

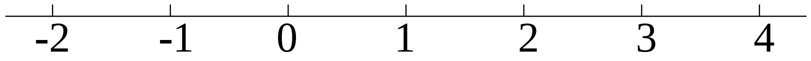
b) Sketch the graph of $h''(x)$ on $[-2, 4]$ as carefully as possible using the axis below.



c) On what intervals is h concave up? On what intervals is h concave down? Briefly explain your assertions.

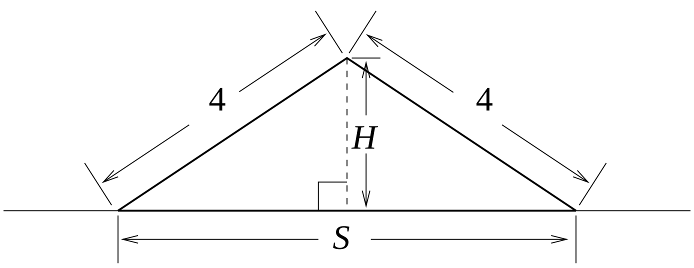
d) For which values of x, if any, does h have relative maxima on $[-2, 4]$? For which values of x, if any, does h have relative mimina on $[-2, 4]$? For which values of x, if any, does h have inflection points on $[-2, 4]$?

e) Use the information you have obtained in previous parts of this problem to sketch a possible graph of $y = h(x)$ using the axis below. Label any relative extrema of h and any inflection points of h on the graph you sketch.



(15) 9. A triangle has two equal sides each 4 units long. What is the length of the third side if the triangle is to have maximum area? Be sure to explain briefly why your answer gives a maximum.

The diagram below may be useful. Use it to express an algebraic relationship between H and $\frac{1}{2}S$ and 4 and then compute the area of the triangle.



(8) 10. Suppose that $F(x) = \frac{1}{101}x^{101} + \frac{5}{37}x^{37} + \frac{8}{5}x^5 + 46x + 8$.

a) What is $F'(x)$?

b) For which x is $F'(x)$ positive? For which x is $F'(x)$ negative?

c) Describe briefly how calculus can be used to verify that $F(77)$ must be bigger than $F(33)$. Do *not* verify this by direct computation of function values.

(12) 11. Suppose $G(x) = -x\cos x + \sin x$.

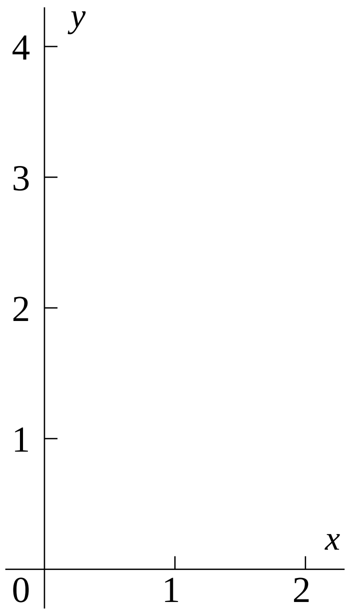
a) Compute $G'(x)$.

b) Use your answer to a) to evaluate $\int_0^{2\pi} x\sin x\,dx$ exactly.

(8) 12. Suppose $k''(x) = e^x - 1$ and $k(0) = 2$ and $k'(0) = -3$. Find a formula for $k(x)$. Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(20) 13. a) Sketch the region in the plane bounded by the $x-$axis, the line $x = 2$, and the graph of $y = \frac{1}{9}x^5$ on the axes given.

b) Find the area of this region using methods of calculus.



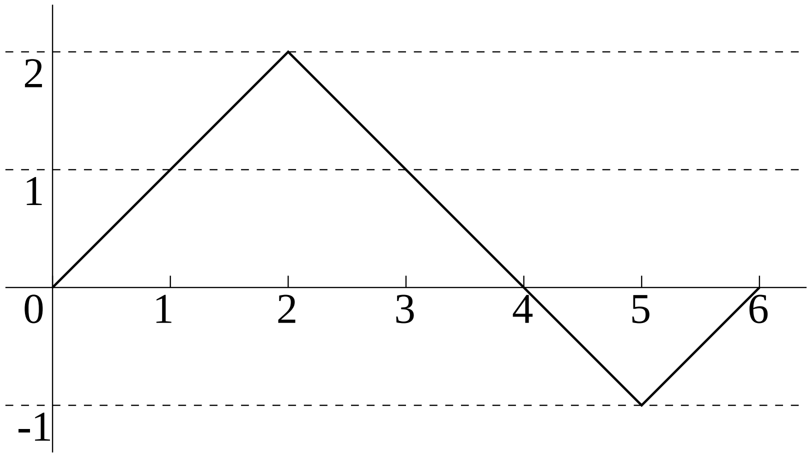
c) Precisely one vertical line with equation $x = c$ will divide this region into two parts with equal area. Use calculus to find this exact value of c.

(16) 14. Evaluate the indicated integrals using methods of calculus.

a) $\int_1^2 \left(3\sqrt{x} - \frac{1}{x^4}\right) dx$

b) $\int (\sin 5x)^3 \cos 5x\,dx$

(10) 15. Below is a graph of the function R which is defined on the interval $[0, 6]$. The graph is made up of three straight line segments.



The graph of R

a) Compute the Riemann sum estimate for $\int_0^6 R(t)\,dt$ associated with the partition $0 < 1 < 2 < 4 < 6$ where the sample points are the midpoints of the subintervals.

b) Compute $\int_3^6 R(t)\,dt$.

c) If Q is the function defined by $Q(x) = \int_0^x R(t)\,dt$, what is $Q'(3)$?

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Math 135 Final Exam

Wednesday, May 6, 1998, from 4 PM to 7 PM

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: __________ LECTURER: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	8	
2	12	
3	10	
4	12	
5	14	
6	15	
7	14	
8	26	
9	15	
10	8	
11	12	
12	8	
13	20	
14	16	
15	10	
Total Points Earned:		

A **A**