Review ADVICE for the final exam in Math 135 (including some review problems)

After this introductory paragraph there are some problems mostly addressing the last 3 or 4 weeks of the course. The final is cumulative and may cover all the material in the course. There will be more emphasis, though, on the material of the last 3 or 4 weeks which has not yet been tested. So please look at the problems below. You may also review the following material, keeping in mind that if we thought it was important earlier, we probably still think it is important.

- The earlier review sheets (with some answers available on the web through the address below).
- The exams that you had earlier in this course.
- Two "real" exams given to one lecture class in this course (with answers and grading guide available on the web through the address below).
- A "real" final exam given last semester in this course (available on the web through the address below).

All this and other Math 135 material can be accessed through:

`http://www.math.rutgers.edu/math135.html`

The final exam will be given **Wednesday, May 6, from 4 to 7 PM** with location to be announced separately for each class.

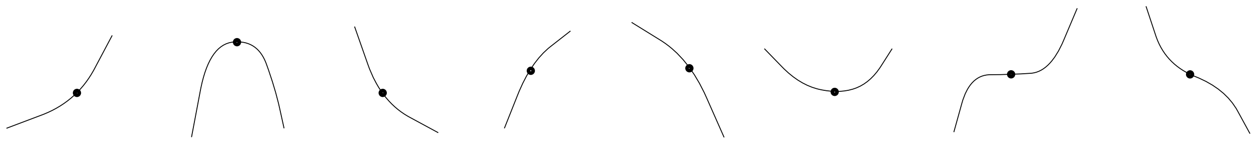
The problems below are intended to be representative of the type of problems which may appear on exams. Please realize that many routine review problems are available in the review sections of the text.

Graphing calculators may be used on the Math 135 final exam but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

1. Suppose Q and its first derivative Q' and its second derivative Q'' have the values indicated in the accompanying table.

Below are some pieces of graphs. Each value of x matches with one picture. Find the matches.

x	$Q(x)$	$Q'(x)$	$Q''(x)$
0	1	-1	-3
1	2	0	7
2	0	-5	2
3	1	0	-2
4	5	2	-2
5	8	-1	0
6	-2	5	3
7	-2	0	0



A B C D E F G H

2. Graph $y = (2x+1)e^{-x^2}$. Use calculus to give information as precisely as possible about local extrema, intervals of increase and decrease, inflection points, and intervals of concavity.

3. The line $y = 3x + 7$ is tangent to the graph of $y = f(x)$ when $x = 4$. What is $f(4)$? what is $f'(4)$?

4. Suppose y is defined implicitly as a function of x by $x^2 + Axy^2 + By^3 = 1$ where A and B are constants to be determined. If the tangent line to this curve has slope -1 at the point $(3, 2)$, find A and B.

5. Evaluate these indefinite integrals.

a) $\displaystyle\int 7x^2 - 3e^x + \frac{5}{x}\,dx$ b) $\displaystyle\int (x^3+5)^2\,dx$ c) $\displaystyle\int 5\sin x + \cos(5x)\,dx$ d) $\displaystyle\int \frac{x}{x^2+5}\,dx$

6. Find a solution to the differential equation $y' = 2x^3 - 1$ passing through the point $(2, 3)$.

7. What's the highest the curve $y = K(x)$ can be if $K''(x)$ is -9 always and if also $K(0) = 0$ and $K(2) = 0$?

8. The following two equations are true:

$$\int_1^4 \frac{x}{(x+1)(x+2)}\,dx = \ln 2 - 2\ln 3 - \ln 5 + 2\ln 6$$

$$\int_2^4 \frac{x}{(x+1)(x+2)}\,dx = \ln 3 - 2\ln 4 - \ln 5 + 2\ln 6$$

Find the rational number B (that is, a quotient of two integers) so that $\displaystyle\int_1^2 \frac{x}{(x+1)(x+2)}\,dx = \ln B$.

9. Evaluate these definite integrals using methods of calculus.

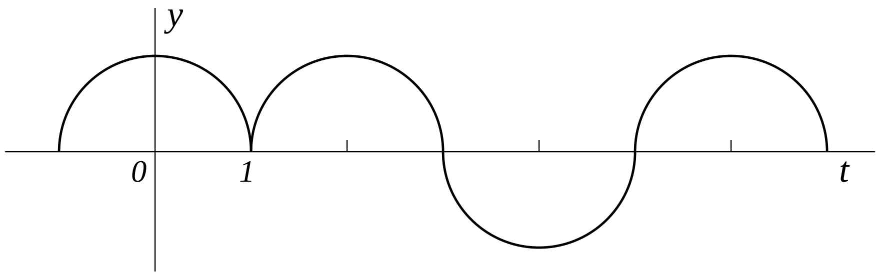
a) $\displaystyle\int_1^2 \left(x^3 - \frac{1}{x^4}\right) dx$ b) $\displaystyle\int_0^{\ln 3} 4e^{2x}\,dx$ c) $\displaystyle\int_0^2 x^2\sqrt{1+3x^3}\,dx$ d) $\displaystyle\int_{\ln \pi}^{\ln 2\pi} e^x \sin(e^x)\,dx$ e) $\displaystyle\int_e^{e^e} \frac{1}{x\ln x}\,dx$

10. a) Compute the derivative of $e^x \sin x - e^x \cos x$.

b) Use your answer to a) to compute $\displaystyle\int_0^{\pi} e^x \sin x\,dx$.

11. If $F(x) = \displaystyle\int_{-42}^{x} \frac{\sin(t^2)}{1+t^4}\,dt$, compute $F(-42)$, $F'(0)$, and $F'(\sqrt{\pi})$ exactly.

12. Below is a graph of $y = f(t)$.



Graph of $y = f(t)$

Note that each segment of the graph is a semicircle. Suppose that $g(x) = \displaystyle\int_{-1}^{x} f(t)\,dt$.

a) What are $g(-1)$ and $g(0)$ and $g(5)$? What are $g'(-1)$ and $g'(0)$ and $g'(5)$?

b) Compute a Riemann sum estimate for $g(3)$ associated with the partition $-1 < 0 < 1 < 3$ where the sample points are the left endpoints of the subintervals.

13. What is the area enclosed by $y = x^5$, the $x-$axis, and the lines $x = 1$ and $x = 2$?

14. Find the area under the arch of $y = x(x-1)^2$. Stated more properly: what is the area of the bounded region in the plane bounded by the $x-$axis and $y = x(x-1)^2$?