Review Problems for the Final Exam in Math 151, Fall 2003

Here are problems written by the principal author (not me!) of this semester's Math 151 final. Please, after you do some problems, check the web, and send me *plain text* e-mail with any solutions of problems which aren't already on the web. Thank you!

1. For what values of B is the following function continuous at $x = 1$?

$$f(x) = \begin{cases} 3x^3 - x^2 + Bx & \text{if } x > 1 \\ Bx - 2 & \text{if } x \le 1 \end{cases}.$$

2. Find the equation of any tangent lines to the curve $y = \frac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.

3. Find the line tangent to the curve defined by the equation $\ln(xy) + 2x - y + 1 = 0$ at the point $(\frac{1}{2}, 2)$.

4. Find the following limits:
a) $\lim\limits_{x\to -1}(x^2 - 2x + 1)$. b) $\lim\limits_{x\to\infty} \frac{3x^2-7}{\sqrt{x^2+2}}$. c) $\lim\limits_{x\to 0} \frac{\sin 3x}{2\sin 5x}$. d) $\lim\limits_{x\to\infty} \frac{3e^x+4e^{-x}}{5e^x+4e^{-x}}$. e) $\lim\limits_{x\to 8^-} \frac{|x-8|}{x-8}$.

5. Find the following limits:
a) $\lim\limits_{x\to 0+} \sqrt{x}\ln x$. b) $\lim\limits_{x\to 0+} x^{\sin x}$. c) $\lim\limits_{x\to\infty} \frac{(\ln x)^2}{x}$. d) $\lim\limits_{x\to\infty} \frac{3e^x+4e^{-x}}{5e^x+4e^{-x}}$.

6. Find y' in each case. Do not simplify your answer.
a) $y = x^7 - 3x + 6 - \frac{1}{x^4}$. b) $y = x^{5x}\sin(x^2)$. c) $y = \frac{2\tan x}{\sqrt{1-x^2}}$. d) $x^4y + 5y^6x^3 = 8$.
e) $xe^{xy+3y} = y$. f) $f(x) = (1+2x)^{1/x}$.

7. State the formal definition of the derivative of the function $f(x)$. Use the definition to calculate $f'(x)$ for $f(x) = \sqrt{3-5x}$.

8. Suppose that $S(x) = \sqrt{x}$ for $x \ge 0$, and suppose f and g are differentiable functions about which the following is known: $f(3) = 2$; $f'(3) = 7$; $g(3) = 4$; $g'(3) = 5$.
Compute: $(f+g)'(3)$; $(f\cdot g)'(3)$; $\left(\frac{f}{g}\right)'(3)$; $(S\circ g)'(3)$; $\left(\frac{f\cdot g}{f-g}\right)'(3)$.

9. Suppose $f''(x) = -3x + \cos(\pi x)$ and $f(1) = 2$ and $f'(1) = -1$. What is $f(5)$?

10. A farmer with 450 feet of fencing wants to enclose the four sides of a rectangular region and then divide the region into four pens with fencing parallel to one side of the rectangle.What is the largest possible total area of the four pens?

11. A ladder which is 13 feet long is leaning against a wall. Its base begins to slide along the floor, away from the wall. When the base is 12 feet away from the wall, the base is moving at the rate of 5 ft/sec. How fast is the top of the ladder sliding down the wall then? How fast is the area of the triangle formed by ladder, wall, and floor changing at that time?

12. Let $f(x) = \frac{3x}{x^2-1}$. Find the function's domain, the intervals where $f(x)$ is increasing or decreasing, any maxima and minima, concavity and inflection points, and any horizontal and vertical asymptotes of the graph of $f(x)$. Then sketch the graph of $f(x)$.

13. Repeat the previous problem with $f(x) = \frac{x^2}{x^2+3}$.

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14. Sketch a graph of $y = f(x)$ if $f(x)$ has the following properties:
$f'(1) = f'(-1) = 0$; $f'(x) < 0$ if $|x| < 1$; $f'(x) > 0$ if $1 < |x| < 2$; $f'(x) = -1$ if $|x| > 2$; $f''(x) < 0$ if $-2 < x < 0$; an inflection point at $(0, 1)$.

15. Find the linearization of $f(x) = \sqrt{x+1}$ at $a = 15$ and use it to find an approximation to $\sqrt{15}$ and to $\sqrt{17}$. Use the second derivative to determine whether the estimate is greater than or less than the actually value.

16. Find the derivative of the following functions:
a) $F(x) = \int_{\pi}^{x} \tan(s^2)\, ds$. b) $G(x) = \int_{1}^{\cos x} \sqrt{1-t^2}\, dt$.

17. Suppose that $f(x)$ is continuous on $[0, 4]$, $f(0) = 1$, and $2 \le f'(x) \le 5$ for all $x \in (0, 4)$. Show that $9 \le f(4) \le 21$.

18. Let $f(x) = 3x^7 - 2x^2 + x - 1$. Show that $f(x)$ must have a real root in $[0, 1]$.

19. Show that the equation $x^{101} + x^{51} + x - 1 = 0$ has exactly one real root.

20. Use two iterations of Newton's method to approximate the root of the equation $x^4 + x - 4 = 0$ in the interval $[2, 3]$ starting with $x_1 = 2$.

21. If a stone is thrown vertically upward from the surface of the moon with an initial velocity of 10 m/s, its height in meters (m) after t seconds (s) is $h(t) = 10t - 0.83t^2$.
a) What is the velocity of the stone after 3 s?
b) What is the maximum height of the stone?

22. Find the absolute maximum and absolute minimum values of $f(x) = \frac{x}{x^2+1}$ in $[0, 2]$.

23. Find the most general antiderivative of these functions:
a) $f(x) = 1 - x^3 + 5x^5 - 3x^7$. b) $g(x) = \frac{5-4x^3+3x^6}{x^6}$ c) $h(x) = 3e^x + 7\sec^2 x + 5(1-x^2)^{-1/2}$.

24. Evaluate these integrals.
a). $\int_1^9 \frac{\sqrt{u}-2u^2}{u}\, du$. b) $\int_0^2 y^2\sqrt{1+y^3}\, dy$. c) $\int_1^5 \frac{dt}{(t-4)^2}$. d) $\int_0^{\pi^{1/3}} t^2 \cos(t^3)\, dt$. e) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$.

25. If $\int_1^5 f(x)\, dx = 12$ and $\int_4^5 f(x)\, dx = 3.6$, find $\int_1^4 f(x) dx$.

26. Find the area bounded by two curves:
a) $y_1 = 2x^2$ and $y_2 = 8x$. b) $y_1 = \sin x$ and $y_2 = \cos x$, $0 \le x \le \frac{\pi}{4}$.

27. What is $\lim\limits_{n\to\infty} \sum\limits_{j=1}^{n} \frac{\pi \sin\left(\frac{j\pi}{n}\right)}{n}$?