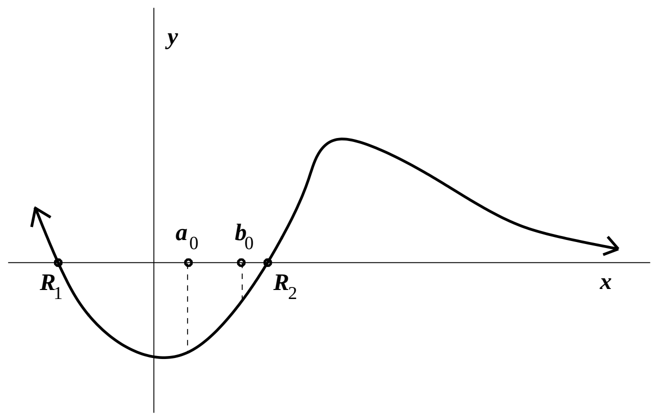
(8) 1. Suppose that $f(x) = \sqrt{17 - x^3}$. Then $f(2) = 3$.

a) Use linear approximation to get an approximate value for $f(1.97)$. You do **not** need to "simplify" your answer!

b) 3.058533472 is the true value of $f(1.97)$ to ten-digit accuracy. Explain briefly using calculus (not calculator evidence!) how you could have predicted that the true value is likely to be greater than or less than the approximate value found in a).

(8) 2. Below is a graph of the function $y = f(x)$. The x−axis is a horizontal asymptote of this graph as $x \to \infty$ and the function f steadily decreases for large x. As $x \to -\infty$ the values of f get larger without any upper bound. Newton's method will be used to approximate a root of $f(x) = 0$.



a) If Newton's method is applied repeatedly with initial value $x = a_0$, a sequence of numbers $\{a_n\}$ is obtained (here a_{n+1} is the result of applying Newton's method to a_n). Draw a_1 and a_2 on the graph above as well as you can. What will happen to a_n as $n \to \infty$?

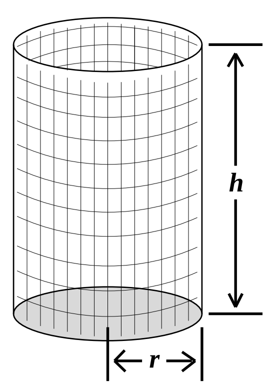
b) If Newton's method is applied repeatedly with initial value $x = b_0$, a sequence of numbers $\{b_n\}$ is obtained (here b_{n+1} is the result of applying Newton's method to b_n). Draw b_1 and b_2 on the graph above as well as you can. What will happen to b_n as $n \to \infty$?

(10) 3. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

(10) 4. Find $f(x)$ if $f''(x) = 2e^x + 3\sin x$ and $f(0) = 0$ and $f'(\pi) = 0$.

(15) 5. A cylindrical can without a top is made to contain V cm^3 of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

Note Be sure you tell why you have found a *minimum*.



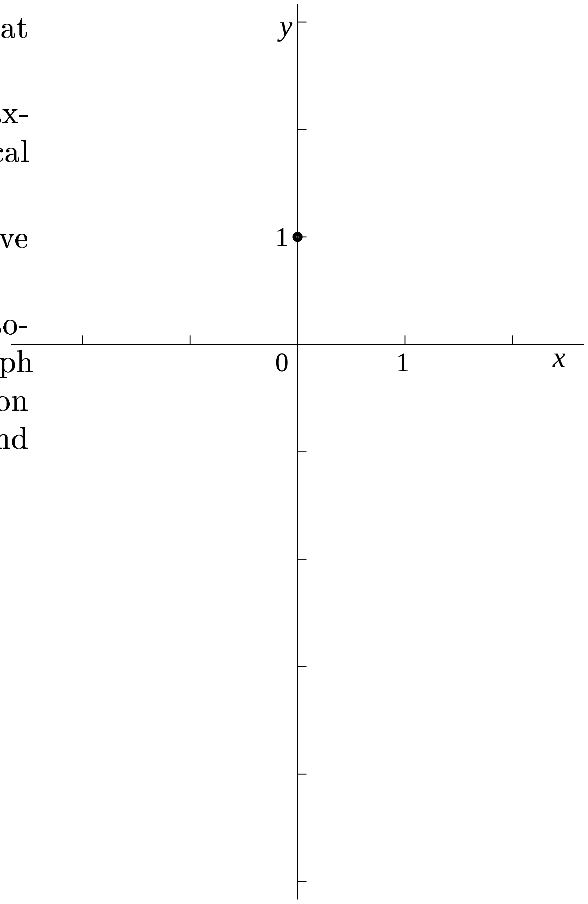
(16) 6. Suppose f is a differentiable function with $f'(x) = (x+1)(x^2-3)$. This is a formula for the *derivative* of f. In the answers to parts a)–c) give exact values, not approximations, using any traditional constants.

a) On what intervals is f increasing and on what intervals is f decreasing?

b) For what values of x does f have a local maximum? For what values of x does f have a local minimum?

c) Compute $f''(x)$. On what intervals is f concave up and on what intervals is f concave down?

d) Use the information given in parts a)–c) together with the fact that $f(0) = 1$ to sketch a graph of $y = f(x)$ on the axes given. Label inflection points with **I**, label local maxima with **M**, and label local minima with **m**.



(10) 7. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

(15) 8. Find the limit, which could be a specific real number or $+\infty$ or $-\infty$ or NOT EXIST. In each case, briefly indicate your reasoning, based on algebra or properties of functions or techniques of the course.

a) $\lim\limits_{x\to\infty} x^3 e^{-x}$

b) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2+x}$

c) $\lim\limits_{x\to\infty} \big(\arctan(x^2) - \arctan(x)\big)$

(8) 9. Find all lines tangent to $y = \dfrac{1}{x}$ which pass through the point $(-4, 2)$.

B B

Second Exam for Math 151

Sections 4, 5, and 6

November 17, 2003

NAME ______________________________

SECTION __________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No texts or notes may be used on this exam.
A formula sheet will be distributed with this exam.
A calculator which does not perform symbolic computation may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	8	
2	8	
3	10	
4	10	
5	15	
6	16	
7	10	
8	15	
9	8	
Total Points Earned:		

B B