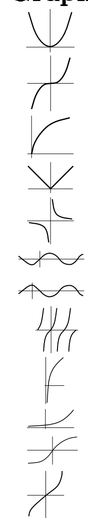
Function	Domain	Range	Graph
x^2	all x	$y \geq 0$	
x^3	all x	all y	
$\sqrt{x}$	$x \geq 0$	$y \geq 0$	
$\lvert x \rvert$	all x	$y \geq 0$	
$1/x$	$x \neq 0$	$y \neq 0$	
$\sin x$	all x	$-1 \leq y \leq 1$	
$\cos x$	all x	$-1 \leq y \leq 1$	
$\tan x$	$x \neq (\text{odd int})\frac{\pi}{2}$	all y	
$\ln x$	$x > 0$	all y	
e^x	all x	$y > 0$	
$\arctan x$	all x	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	
$\arcsin x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$	



If $\lim_{x \to a^{(\pm)}} f(x) = \pm\infty$ then $x = a$ is a **vertical asymptote** of $y = f(x)$ and if $\lim_{x \to \pm\infty} f(x) = b$ then $y = b$ is a **horizontal asymptote** of $y = f(x)$.

Function	Derivative
C	0
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$1/x$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$(\sec x)^2$
$\arctan x$	$\frac{1}{1+x^2}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$

Function	Derivative
$Kf(x)$	$Kf'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$

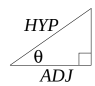
Logarithmic properties

$\ln(a \cdot b) = \ln a + \ln b$ $\ln(a^b) = b\ln(a)$
$\ln(a/b) = \ln(a) - \ln(b)$ $\ln(\frac{1}{b}) = -\ln(b)$
$\ln(e^a) = a$ $\ln(1) = 0$ $\ln(e) = 1$

Exponential properties

$a^{b+c} = a^b \cdot a^c$ $a^{-b} = 1/a^b$
$(a^b)^c = a^{bc}$ $e \approx 2.718$
$a^0 = 1$ $e^{\ln a} = a$ if $a > 0$

Triangle things 360^o (degrees) $= 2\pi$ radians



$\sin\theta = \frac{OPP}{HYP}$
$\cos\theta = \frac{ADJ}{HYP}$
$\tan\theta = \frac{OPP}{ADJ}$

Pythagoras

$(ADJ)^2 + (OPP)^2 = (HYP)^2$
$(\sin\theta)^2 + (\cos\theta)^2 = 1$

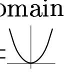
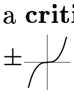
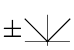
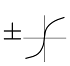
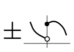
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	*NONE*
π	0	-1	0

More formulas

The **roots** of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.
Distance from (a, b) to (c, d): $\sqrt{(a-c)^2 + (b-d)^2}$
Circle center (h, k) & radius r: $(x-h)^2 + (y-k)^2 = r^2$
Line $y = mx + b$ and $m = \frac{y_2 - y_1}{x_2 - x_1}$ (**slope** of the line)
Addition formulas $\sin(A + B) = \sin A \cos B + \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
Periodicity $\sin(x + 2\pi) = \sin x$ and $\cos(x + 2\pi) = \cos x$ and $\tan(x + \pi) = \tan x$ for all x

Area and Volume Formulas

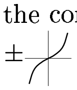
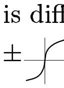
Triangle $A = \frac{1}{2}$ BASE·HEIGHT
Rectangle $A =$ LENGTH·WIDTH
Circle $A = \pi$ RADIUS^2
Circle $C = 2\pi$ RADIUS
Box $V =$ LENGTH·WIDTH·HEIGHT
Cylinder $V = \pi$ RADIUS^2·HEIGHT
Cone $V = \frac{1}{3}\pi$ RADIUS^2·HEIGHT
Sphere $A = 4\pi$ RADIUS^2
Sphere $V = \frac{4}{3}\pi$ RADIUS^3

w in f's domain is a **critical number** if **either** $f'(w) = 0$: could look like $\pm$  or $\pm$  **or** $f'(w)$ doesn't exist: might look like $\pm$  or $\pm$  or even like $\pm$  if f isn't continuous at w.

Finding max/min on a closed interval If f is continuous on $a \leq x \leq b$ then f's max/min values must occur either at a or at b or at a critical number inside the interval.

The First Derivative Test A critical number w is a **relative max** if $f'(\text{left of } w) > 0$ & $f'(\text{right of } w) < 0$; **relative min** if $f'(\text{left of } w) < 0$ & $f'(\text{right of } w) > 0$.
Important No other critical numbers should be between w and where the sign of f' is checked!
If both are positive or both are negative, then w is an **inflection point** of f.

The Second Derivative Test A critical number w is a **relative min** if $f''(w) > 0$ & **relative max** if $f''(w) < 0$.

f has an **inflection point** at w if w is in f's domain and if the concavity of f's graph is different on either side of w: $\pm$  (here $f''(w) = 0$) or $\pm$  (here $f''(w)$ doesn't exist).

f is **continuous** at w if $\lim_{x \to w} f(x)$ exists and equals $f(w)$ **or** check $\lim_{x \to w^+} f(x)$ and $\lim_{x \to w^-} f(x)$ both exist and $= f(w)$.
f is **differentiable** at w if $\lim_{h \to 0} \frac{f(w+h) - f(w)}{h}$ exists. This is $f'(w)$: **the rate of change of f with respect to w** or **the slope of the tangent line** to $y = f(x)$ at $x = w$.

Implicit differentiation/related rates
Key point Differentiate a whole equation. Don't forget what's varying, chain rule, product rule, etc.
Example If $xy^2 = \sin(x + y) + 3x$ then $\frac{d}{dx}$ the equation. Get $1 \cdot y^2 + x \cdot 2yy' = \cos(x+y)(1+y') + 3$. **Solve** for y'.

OVER

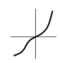
f defined in $a < x < b$ has a **relative maximum** at w in the interval if $f(w) \geq f(x)$ for x's near w on both sides. f defined in $a < x < b$ has a **relative minimum** at w in the interval if $f(w) \leq f(x)$ for x's near w on both sides. Relative max and min must occur **at critical numbers**.

Differential or tangent line approximation $f(x+h) \approx f(x) + f'(x)h$. The graph's bending causes **error**: the true value is larger when the graph is concave up and smaller when the graph is concave down.

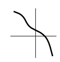
Intermediate Value Theorem If f is continuous in $a \leq x \leq b$, f's values include all numbers between $f(a)$ and $f(b)$: a continuous function's graph has no jumps. **Mean Value Theorem** If f is differentiable in $a \leq x \leq b$, there are some c's in the interval with $f'(c) = \frac{f(b)-f(a)}{b-a}$: some tangent lines of a differentiable function's graph must be parallel to any chord. **Rolle's Theorem** MVT with $f(a) = f(b) = 0$.

Function	**Antiderivative**
$f(x)$	$F(x) + C$
$Kf(x)$	$KF(x)$
$f(x) + g(x)$	$F(x) + G(x)$
x^n	$\frac{1}{n+1}x^{n+1} + C$, $n \neq -1$
$\frac{1}{x}$	$\ln x + C$ $(x > 0)$
e^x	$e^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\tan x$	$\ln(\sec x) + C$

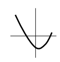
f is **increasing** in $a < x < b$ if $f(x_1) \leq f(x_2)$ for any $x_1 \leq x_2$ in the interval. If $f'(x) > 0$ always in $a < x < b$ then f is increasing there.



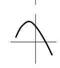
f is **decreasing** in $a < x < b$ if $f(x_1) \geq f(x_2)$ for any $x_1 \leq x_2$ in the interval. If $f'(x) < 0$ always in $a < x < b$ then f is decreasing there.



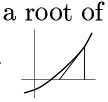
f is **concave up** if lines connecting the graph are above the graph: it bends *up*. If $f''(x) > 0$ always in $a < x < b$, f is concave up.



f is **concave down** if lines connecting the graph are below the graph: it bends *down*. If $f''(x) < 0$ always in $a < x < b$, f is concave down.



Newton's method
A way to improve a guess for a root of $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.



L'Hopital's Rule
A way to evaluate certain limits: if $\lim_{x \to a} \frac{f(x)}{g(x)}$ has the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and if $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

OVER