Useful information for the first exam in Math 151:04–06, Fall 2003

The time, date, and place will be:

Hill Center 116, Monday, October 13, at 1:10 PM

The exam will cover material up to and including section 3.6 of the text (Lecture 10). Some of the emphasis of the class meetings has been different from what's in the text.

Review Session Sunday, October 12, at 4:30 in the afternoon, probably in our usual classroom, Hill 116.

Some rules for the exam:

• No books or notes or calculators.

• Please leave answers in "unsimplified" form: $15^2 + (.07) \cdot (93.7)$ is preferred to 231.559. You should know simple exact values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$. Math constants such as π and e should be left "as is" and not approximated.

• Show your work: an answer alone may not receive full credit.

Almost all of the problems below came from previous Math 151 exams. The total number of questions (problems and parts of problems) is more than would be given on a real exam.

1. Note that $(x-1)(2x-3) = 2x^2 - 5x + 3$.

a) Find an equation for the line tangent to $y = 2x^2 - 5x + 3$ when $x = -1$.

b) Sketch the line found in a) and the curve $y = 2x^2 - 5x + 3$ on the same axes.

2. Suppose that $S(x) = \sqrt{x}$ for $x \geq 0$, and that f and g are differentiable functions about which the following is known:

$$f(3) = 2 \qquad f'(3) = 7 \qquad g(3) = 4 \qquad g'(3) = 5\,.$$

Carefully compute the following quantities (an answer alone will *not* receive full credit):

$$(f+g)'(3) \qquad (f \cdot g)'(3) \qquad \left(\frac{f}{g}\right)'(3) \qquad (S \circ g)'(3)\,.$$

3. Suppose that $f(x) = \dfrac{|x^2-1|}{x-1}$.

a) Find

i) $\lim\limits_{x \to 1^-} f(x)$ ii) $\lim\limits_{x \to 1^-} f(x)$ iii) $\lim\limits_{x \to -1^-} f(x)$ iv) $\lim\limits_{x \to -1^+} f(x)$

b) Does $\lim\limits_{x \to 1} f(x)$ exist? Does $\lim\limits_{x \to -1} f(x)$ exist?

c) Sketch the graph of $y = f(x)$.

4. a) State the formal definition of the derivative, $f'(x)$ of the function $f(x)$.

b) Use your answer to a) combined with algebraic manipulation and standard properties of limits to compute the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$.

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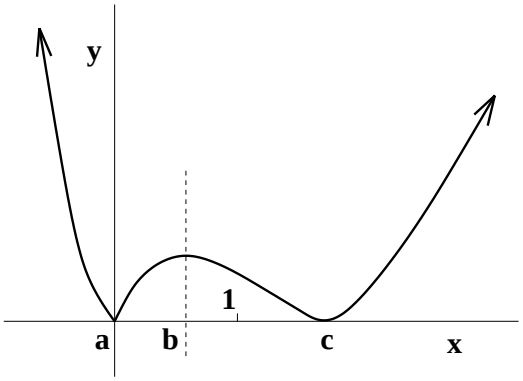
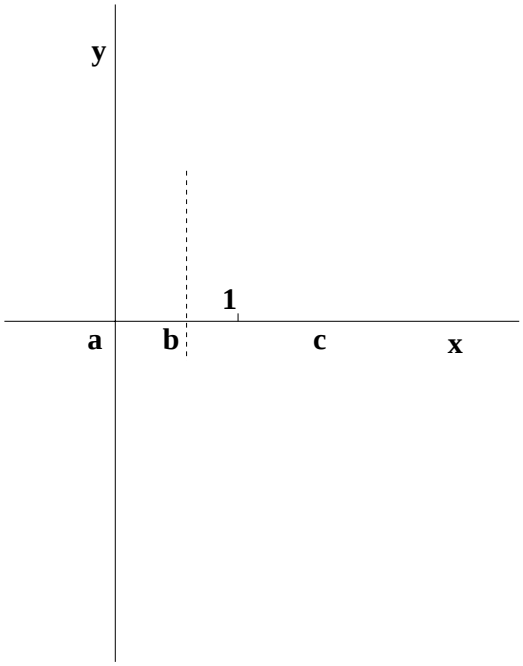
5. We saw in class that the graphs of sine and cosine do *not* intersect perpendicularly. Find a positive number A so that $y = A\sin x$ and $y = A\cos x$ *do* intersect perpendicularly between 0 and $\frac{\pi}{2}$.

6. Use complete English sentences to explain why the following statements are true. If you use a theorem from the course, explain why the hypotheses of the theorem are satisfied.

> Suppose $f(x) = e^{-x} + \sin(Bx) + B$ (note that the parameter B appears in two places in the formula for f). If $B = -\frac{1}{2}$, the equation $f(x) = 0$ must have at least one positive root. If $B = -10$, the equation $f(x) = 0$ has no positive root.

7. On the left is the graph of $y = f(x)$. Sketch as well as you can the graph of $y = f'(x)$ on the axes provided on the right.





8. All answers to this question should be one of the following:

A specific real number (such as 2 or $\sqrt{3}$, but arithmetic operations may be left unsimplified: thus $5+7^2$ is an allowable answer) **or** $+\infty$ **or** $-\infty$ **or** if none of the previous answers applies, write "Does not exist."

a) $\lim_{x\to -1} x^2 - 2x + 1$ b) $\lim_{x\to\infty} \dfrac{3x^2-7}{\sqrt{x^2+2}}$ c) $\lim_{x\to 2} \dfrac{2-x}{\sqrt{2}-\sqrt{x}}$ d) $\lim_{x\to 5} \dfrac{x^2-2x-15}{x^2+x-30}$

e) $\lim_{x\to -5^-} \dfrac{x}{x+5}$ f) $\lim_{x\to 4} \dfrac{x^2+1}{(x-4)^3}$ g) $\lim_{x\to\infty} \dfrac{3e^x+4e^{-x}}{5e^x+6e^{-x}}$ h) $\lim_{x\to -\infty} \dfrac{3e^x+4e^{-x}}{5e^x+6e^{-x}}$

9. Find $\dfrac{dy}{dx}$ in each case. Do **not** simplify your answers in any way. For example, the derivative of $37x^{46}$ should be written as $(46)37x^{45}$.

a) $y = x^7 - 3x + 6 + \dfrac{1}{x^4}$ b) $y = e^{5x}\sin 2x$ c) $y = \dfrac{2\tan x}{5-x}$ d) $y = \left(x^4+5x+5\right)^{1/5}$

e) $y = \cos\left(\sqrt{9x^2+1}\right)$ f) $x^4y + 5y^6x^3 = 8$ g) $xe^{xy+3y} = y$.