Useful information for the second exam in Math 151:04–06, Fall 2003

The time, date, and place are **Hill Center 116, Monday, November 17, at 1:10 PM**.

The exam will be cumulative but will emphasize what we've done since the first exam: the end of chapter 3 and the material we will have covered in chapter 4 (up to Lecture 20). Some of what's been done in class has been different from the contents of the text.

Review Session Sunday, November 16, at 4:30 in the afternoon, in Hill 116.

Some rules for this exam *only*:

• You may use any calculator which does *not* have symbolic computation capabilities. No calculator will be permitted on the final exam!

• Please leave answers in "unsimplified" form: $15^2 + (.07) \cdot (93.7)$ is preferred to 231.559. You should know simple exact values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$. Math constants such as π and e should be left "as is" and not approximated.

• Show your work: an answer alone may not receive full credit.

• A formula sheet will accompany the exam. A *draft* of the formula sheet is linked to the course home page and can be inspected. Please send comments if you would like.

Almost all of the problems below came from previous Math 151 exams. The total number of questions (problems and parts of problems) is more than would be given on a real exam.

1. Suppose $B(x)$ is a differentiable function with $B(2) = 3$ and $B'(x) = \sqrt{23 - 7x}$ for all real numbers x. Suppose also that $C(x) = 5x^2 - 3$. Let $A(x) = B(C(x))$.

a) Compute $A(1)$. Write a formula for $A'(x)$ only in terms of x and then find $A'(1)$.

b) Use your answers to a) to write an equation for the line tangent to $y = A(x)$ when $x = 1$. YOU NEED NOT SIMPLIFY THE ANSWER!

c) Use your answers to a) and linearization to find an approximate value of $A(0.95)$. YOU NEED NOT SIMPLIFY THE ANSWER!

d) It is true that $A''(1) = -\frac{260}{3}$. Is the estimate you found in c) likely to be greater than or less than the true value of $A(0.95)$? Give a brief reason supporting your answer.

2. In this problem, $f(x) = (x^2 - 1)e^x$.

a) Use your calculator to sketch the graph of $y = f(x)$ when $-3 \le x \le 3$ and $-5 \le y \le 5$.

b) Verify the geometric properties observed in the graph using the analytic tools of calculus:

• What are $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ and why? Explain how your results apply to the graph sketched.

• Compute $f'(x)$. Find the exact roots of $f'(x) = 0$ algebraically. Where is $f'(x)$ positive and where is it negative? Explain how your results apply to the graph sketched.

• Compute $f''(x)$. Find the exact roots of $f''(x) = 0$ algebraically. Where is $f''(x)$ positive and where is it negative? Explain how your results apply to the graph sketched.

3. a) Suppose $f(x) = x^5 + 3x + \sin(x - 5) + 2$. Explain carefully using complete English sentences why the statement "If $0 < x < 1$, then $f(0) < f(x) < f(1)$" is correct. Using calculator evidence is *not* acceptable here! You may need a result from the course. Explain why it is applicable. Estimate (you should use your calculator) $f(0)$ and $f(1)$.

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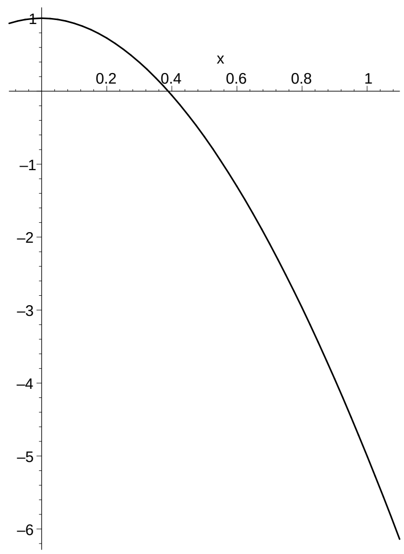
b) Now suppose that $F'(x) = f(x)$, with $f(x)$ as defined in a). Use your calculator and the results of a) to answer the following question: if you know that $F(0) = 3$, how big and how small can $F(1)$ be? Explain your reasoning and your computation briefly using complete English sentences.

4. Suppose $f(x) = (1+2x)^{1/x}$.

a) Use your calculator to compute $f(.002)$ and $f(-.003)$.

b) Find $\lim_{x\to 0} f(x)$. GIVE AN EXACT VALUE, USING ANY TRADITIONAL CONSTANTS.

5. Suppose $f(x) = x^3 - 7x^2 + 1$. $f(0) = 1$ and $f(1) = -5$, so $f(x) = 0$ has a solution in the interval $0 \le x \le 1$. On the right is a graph of $y = f(x)$ on this interval. We will use Newton's method to solve $f(x) = 0$.



a) Suppose we begin Newton's method with an initial guess of $x_0 = 1$. Draw the next two approximations x_1 and x_2 on the graph shown as well as you can, including any useful tangent lines.

b) Write the specific algebraic iteration for this $f(x)$ describing how Newton's method goes from x_n to x_{n+1}.

c) Use your calculator to give approximate values of x_1 and x_2, the first two iterates of Newton's method for solving $f(x) = 0$ with initial guess $x_0 = 1$.

8. Find the line tangent to the curve defined by the equation $\ln(xy) + 2x - y + 1 = 0$ at the point $\left(\frac{1}{2}, 2\right)$.

9. Ant **A** is crawling up a vertical pole at .3 meters/minute. At the same time ant **B** is crawling away from the base of the pole on the horizontal ground at .4 meters/minute.

a) If θ is the angle that ant **B** sees between the base of the pole and ant **A**, if a is the distance from ant **A** to the base of the pole, and if b is the distance from ant **B** to the base of the pole, express θ as a function of a and b.

b) Use the information provided to compute θ and $\frac{d\theta}{dt}$ at the instant that ant **A** is 10 meters up the pole and ant **B** is 5 meters from the base of the pole.

10. Find the limit, which could be a specific real number or $+\infty$ or $-\infty$. In each case, briefly indicate your reasoning, based on algebra or properties of functions.

a) $\lim_{x\to\infty} \frac{\ln(\ln(x))}{(\ln(x))^2}$ b) $\lim_{x\to 3} \frac{x^2-5x+6}{x+3}$ c) $\lim_{x\to 0} \frac{\sin(x)-x}{(e^x-1)^2}$

11. A farmer with 450 feet of fencing wants to enclose the four sides of a rectangular region and then divide the region into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

12. Suppose $f''(x) = -3x + \cos(\pi x)$ and $f(1) = 2$ and $f'(1) = -1$. What is $f(5)$?