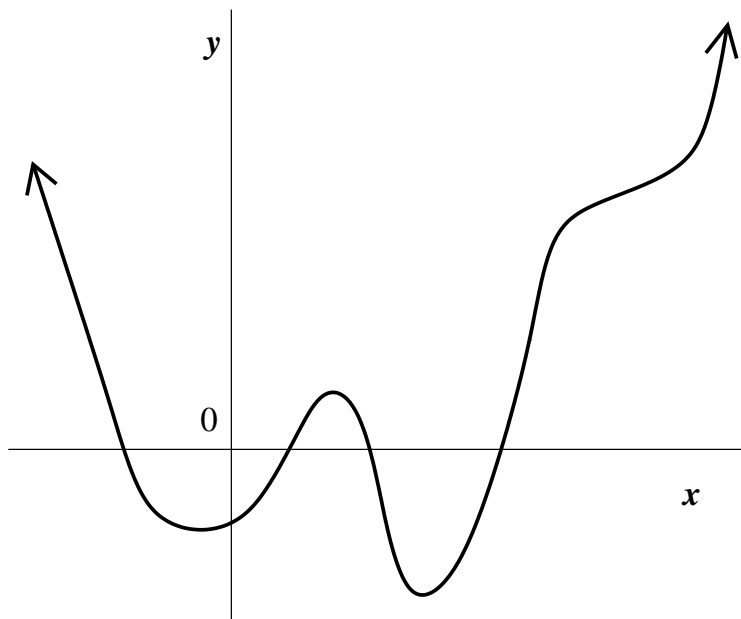


1. This picture is the graph of a polynomial. What is the lowest degree that this polynomial could have? Explain your answer.



2. A charged particle moves along the x -axis under the influence of an electric field. The field strength varies with time so that the velocity of the particle is complicated. Let the position of the particle at time t be denoted by $x = x(t)$, and the velocity of the particle at time t be denoted $v = v(t)$.

Suppose $x(0) = 0$, and

$$v(t) = \begin{cases} 2t - 1, & \text{if } 0 \leq t \leq 1 \\ 4t - 3, & \text{if } 1 \leq t \leq 2 \\ 6t - 7, & \text{if } 2 \leq t \leq 3 \end{cases}$$

What is $x(1)$? What is $x(2)$? What is $x(3)$? Sketch the graphs of $x = x(t)$ and $v = v(t)$.

3. Define $f(x)$ on $0 \leq x \leq 1$ by: $f(x)$ is the first digit in the decimal expansion for x . For example, $f(.5) = 5$ and $f(0.719) = 7$.

a) Sketch the graph of $y = f(x)$. Use appropriate scales for x and for y .

b) Use the geometric definition of the definite integral to compute $\int_0^1 f(x) dx$.

c) Define $g(x)$ by: $g(x)$ is the second digit in the decimal expansion for x . For example, $g(0.437) = 3$. Compute $\int_0^1 g(x) dx$.

d) Which is larger, $\int_0^1 x^2 f(x) dx$ or $\int_0^1 x^2 g(x) dx$?

OVER

4. Water is flowing through mighty Mill Brook (a local stream located only a short distance from where this was written). Two engineering students (Alpha and Beta) are assigned the task of estimating the water flow through the brook between 8 AM and noon. They are provided with instruments which will allow them to tell what the water flow is at any time in **gallons per minute**.

a) Lazy student Alpha makes observations of the water flow every hour on the hour from 8 AM to noon. Here are Alpha's observations:

Time	Flow, in Gals/Min	
8:00	5.6	
9:00	8.9	(Thunderstorm!)
10:00	12.4	
11:00	7.2	(Lunch break.)
12:00	

What is Alpha's approximation for the total water drained through Mill Brook during those 4 hours?

b) Student Beta, more energetic, checks the water flow every *half* hour. Here's Beta's data:

Time	Flow, in Gals/Min	
8:00	5.6	
8:30	7.3	
9:00	8.9	
9:30	8.3	(Thunderstorm!)
10:00	12.4	
10:00	11.2	
11:00	7.2	
11:00	6.1	(Lunch break.)
12:00	

What is Beta's estimate of the total water flow through Mill Brook during the period 8 AM to noon?