1. a) Sketch a graph of $y = \big|x - |x-3|\big|$.

b) Give a "piecemeal" definition of the function whose graph was sketched in a) *without* mentioning absolute value. The graph may help to answer this question, but you should justify your answer algebraically with a case-by-case argument from the equation for y (of the sort "when $x \geq 3$ then y is given by the formula ...").

2. Let $f(x) = \dfrac{\tan x - x}{x^3}$ for $x \neq 0$.

a) Graph $y = f(x)$ in the window $-0.5 \leq x \leq 0.5$. Does your graph suggest that $\lim_{x\to 0} f(x)$ exists? What seems to be the value of the limit?

b) Graph $y = f(x)$ in the window $-10^{-5} \leq x \leq 10^{-5}$. Do you still think that $\lim_{x\to 0} f(x)$ exists?

In fact the limit does exist, and the value is correctly predicted by a), but you can't prove this, and b) should make you suspicious. We will verify this limit later using a result called l'Hopital's rule.

3. a) The function S (the "squaring function") has domain all of $\mathbb{R}$ (all real numbers) and its values are given by $S(x) = x^2$ for all x. Now consider the function T whose domain is also all of $\mathbb{R}$ and which is defined by

$$T(x) = \begin{cases} S(x) & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}$$

Sketch a graph of T. What is $\lim_{x\to 5} T(x)$ and what is $\lim_{x\to 3} T(x)$? Give some reason to believe your assertions.

b) Suppose S is again the squaring function defined above. Now an evil interstellar visitor comes and *changes* exactly *one million values* of S and thus creates a new function, V. What can you say about $\lim_{x\to a} V(x)$ for all values of a? Give reasoning to support your assertions.

4. A bug named Fred crawls up the y-axis at constant speed so that its (his?) position at time t (t here is a positive number) is $(0,t)$. Also another bug named Jane (!) crawls on the curve $y = \sqrt{x}$ so that the first coordinate of Jane's position at time t is t.

a) Draw a picture showing the bugs at time $t = 4$. Also draw the line segment connecting the positions of the bugs at that time. What is the slope of this line segment?

b) Draw a picture showing the bugs at time $t = 100$. Also draw the line segment connecting the positions of the bugs at that time. What is the slope of this line segment?

c) What happens when t gets very large positive? What happens to the length of the line segment, and what happens to the slope of the line segment? Explain your statements as well as you can. Use appropriate drawing and algebraic analysis of the situation.