

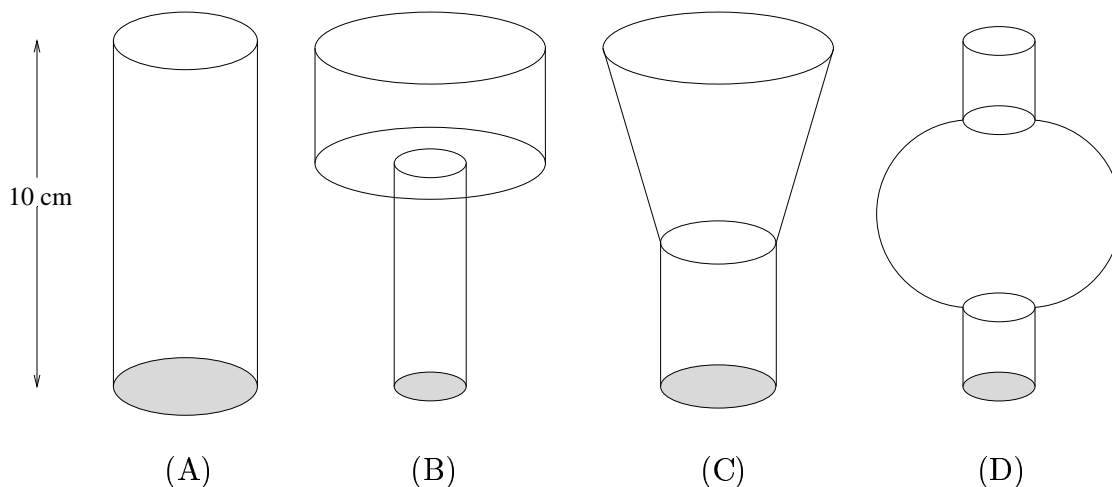
1. a) Suppose $f(x) = \sqrt{x^2 + 1} - x$. What is $\lim_{x \rightarrow \infty} f(x)$?

b) Suppose $g(x) = x$. What is $\lim_{x \rightarrow \infty} g(x)$?

c) Suppose $h(x) = f(x) \cdot g(x)$. What is $\lim_{x \rightarrow \infty} h(x)$?

2. If the line $13x - 8y + 5 = 0$ is tangent to the graph of $y = F(x)$ when $x = 7$, what are $F(7)$ and $F'(7)$?

3. Four containers are each 10 cm tall, and each of them has a volume of 30 cm^3 . They are each being filled by a liquid at the rate of 5 cm^3 per minute. Here is a picture of the four containers:



a) For each of the containers, graph the height, $h(t)$, of the level of the liquid in the containers measured in centimeters as a function of time, t , measured in minutes.

b) Which of the functions graphed in a) are continuous? Justify your conclusions as well as you can.

c) Which of the functions graphed in a) are differentiable? Justify your conclusions as well as you can.

4. a) Show from the definition of derivative that

i) $f(x) = x|x|$ is differentiable at $x = 0$.

ii) $g(x) = (1 + |x|)^{1/2}$ is *not* differentiable at $x = 0$.

Note The most direct way to do this is to consider limits from “both sides” separately: $\lim_{x \rightarrow 0^-}$ and $\lim_{x \rightarrow 0^+}$. Then the definition of $||$ to complete your analysis.

b) Graph each function carefully near $x = 0$ and discuss how the graphs appear to confirm the results of a).