1. Two long sections of road are represented by two parallel half lines: one, by $y = 0$ and $x \leq 0$ and the other, by $y = 1$ and $x \geq 1$. The roads must be connected by a curve matching values and first and second derivatives at both of the connecting points, $(0,0)$ and $(1,1)$. Then a car traveling on the road at constant speed will not be jolted by sharp transverse forces at the connecting points (since force and acceleration are proportional, and acceleration is second derivative). Find a function $y = f(x)$ which is at least twice differentiable so that the requirements will be fulfilled. Sketch a graph of this function.

2. a) Compute $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$ and $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$.

b) Compute $\lim\limits_{x \to 0} \dfrac{e^{5x} - 1}{3x}$ and $\lim\limits_{x \to 0} \dfrac{\sin 7x}{6x}$.

c) Compute $\lim\limits_{x \to 0} \dfrac{\left(\sin(4x)\right)^2 \sin 7x}{\left(e^{3x} - 1\right)^2 x \left(\cos(5x)\right)}$.

3. Can two differentiable functions f and g satisfy all of the following conditions at once: $f(0) = 0$, $g(0) = 0$, and $f(x)g(x) = x$ for all x?

4. Consider the curves $y = x^2$ and $y = C - x^2$ (here C is a constant to be precisely determined later).

a) Sketch $y = x^2$ and $y = C - x^2$ for $C = -1$, $C = 1/2$, and $C = 2$ on the same axes.

b) Find a value of C so that the curves $y = x^2$ and $y = C - x^2$ have two intersection points, and so that, at the intersection points, the lines tangent to the two curves are perpendicular. Sketch the curves and the tangent lines for this particular value of C.
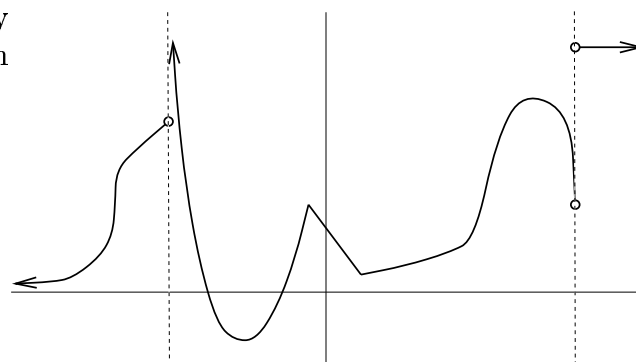
Note Such pairs of curves are important in the study of both electromagnetism and heat transfer. They are called *orthogonal curves.*

5. Here is the graph of a function, F. Label any points on the graph which occur in your discussion of the questions below.

a) Where is F not continuous? Why?

b) Where is F not differentiable? Why?

c) Sketch a graph of F' wherever F' is defined.

Graph of $y = F(x)$

Part of this problem is coping with the person* who didn't label the axes or the units!

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* idiot?