1. Suppose $f(x) = \arctan x + \arctan\left(\dfrac{1}{x}\right)$.

a) Compute $f'(x)$ carefully and *simplify* it as much as you can.

b) What does the Mean Value Theorem state about a function whose derivative is what you got in a)? (See the textbook or your notes if you don't remember!)

c) Now graph f on your graphing calculator in the window $-4 \le x \le 4$ and $-2 \le y \le 2$. Compare what the picture shows with the answer to b) and explain the difference.

2. a) Suppose $P(x)$ is any polynomial of degree four which has four distinct real roots. Can $P'(x)$ have fewer than three real roots? Why or why not? How many real roots does $P''(x)$ have?

b) Give an example of a polynomial of degree four which has only three distinct real roots. How many real roots does its derivative have?

3. Suppose you **know** that $g'(x) = \dfrac{2}{1+x^4} - \dfrac{3}{4+x^4}$. Is $g(0) < g(1)$?

Note #1 It is not likely at this time that you can write a formula for a g with this derivative (that can be done, and such g's have very complicated formulas). So you will have to make some *indirect* argument, just using the information you have about g'. Write out **two verifications** of your answer, one an algebraic argument using the formula for g and the other, a geometric argument, using a graph of g (which can be plotted on a calculator).

Note #2 Here is such a function:

$$g(x) = \frac{\sqrt{2}}{4}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right) + \frac{\sqrt{2}}{2}\arctan(\sqrt{2}x+1) + \frac{\sqrt{2}}{2}\arctan(\sqrt{2}x-1)$$
$$+\frac{3}{16}\ln(x^2-2x+2) - \frac{3}{8}\arctan(x-1) - \frac{3}{16}\ln(x^2+2x+2) - \frac{3}{8}\arctan(x+1)$$

Does this formula, which should be checked if it is used, help, or is studying the derivative easier?

4. a) Suppose you know that $h'(x) = (x-1)(x-2)^2(x-3)^3(x-4)^4(x-5)^5$. What are the critical numbers of h? Which of them are local extrema, and what kind of local extrema are they?

b) Suppose you know that $k'(x) = x(x-1)^{2/3}(x-2)^{3/5}(x-3)^{4/7}$. What are the critical numbers of k? Which of them are local extrema, and what kind of local extrema are they?

Note: you are *not* asked to compute h and k explicitly.