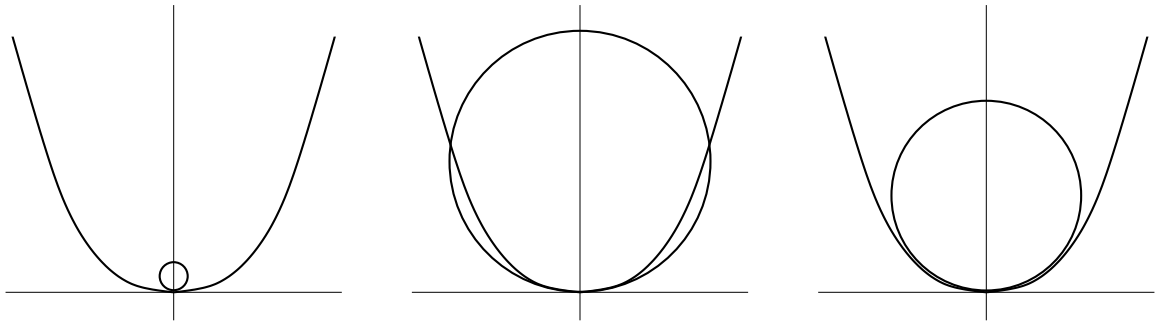
1. Suppose $5x^3y - 3xy^2 + y^3 = 6$. The point $(1, 2)$ is on this curve. Is the curve concave up or concave down at $(1, 2)$?

Explicit way to go y *can* be solved as a function of x.* Then you can differentiate the formula twice and evaluate when $x = 1$.

Implicit way to go Find $\dfrac{dy}{dx}$ implicitly and then differentiate again to get $\dfrac{d^2y}{dx^2}$. Evaluate everything at $(1, 2)$.

2. Find the magical circle which is largest and which just fits "inside" the parabola $y = x^2$.



Too small! Too large! Just right.

The parabola $y = x^2$ and some inside circles tangent at $(0, 0)$

3. a) Compute $(1.5)^{1/(1.5)}$, $2^{1/2}$, $(2.5)^{1/(2.5)}$, $3^{1/3}$, and $(3.5)^{1/(3.5)}$.

b) Now consider the function $V(x) = x^{1/x}$, whose domain is $(0, \infty)$. Compute $V'(x)$ and find any critical points of V. Either compute V'' or study V' more closely to conclude information about the nature of the critical point(s?) of V.

c) Compare your results from a) and b).

d) What happens to $V(x)$ as $x \to \infty$? What happens to $V(x)$ as $x \to 0^+$? Carefully evaluate these limits using appropriate methods from the course.

e) How many inflection points do you think that the graph of $y = V(x)$ has?

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* Here it is (really!):

$$y = \left(-\frac{5}{2}x^4 + 3 + x^3 + \frac{1}{18}\sqrt{1500x^9 - 675x^8 - 4860x^4 + 2916 + 1944x^3}\right)^{1/3} - \frac{\frac{5}{3}x^3 - x^2}{\left(-\frac{5}{2}x^4 + 3 + x^3 + \frac{1}{18}\sqrt{1500x^9 - 675x^8 - 4860x^4 + 2916 + 1944x^3}\right)^{1/3}} + x\,.$$

Does this help?