1. A program called *Macsyma* can find many indefinite integrals. It is a bit old, but it can do 4 of the following 5 integrals in a total of less than 5 seconds of CPU time. Please do 3 of these integrals. (You choose which ones!)

a) $\int x \sin(x^2) \, dx$ b) $\int x \, (\sin x)^2 \, dx$ c) $\int x^2 \sin x \, dx$

d) $\int x^2 \, (\sin x)^2 \, dx$ e) $\int x^2 \sin(x^2) \, dx$

2. a) When D is very large, show that the integral $\int_1^2 e^{-Dx} \, dx$ is very small. (You may wish to draw a picture, but other verification is also necessary.)

b) When D is very large, show that the integral $\int_1^2 xe^{-Dx} \, dx$ is very small. (You may wish to draw a picture, but other verification is also necessary.)

c) When D is very large, show that the integral $\int_1^2 \frac{1}{1 + 5x^{48}} e^{-Dx} \, dx$ is very small*. (You may wish to draw a picture, but other verification is also necessary.)

3. Consider the function $G(x) = e^x \sin Nx$ on the interval $[0, 1]$.

a) With a sketch or otherwise, describe this function when $N = 3$ and when $N = 10$ and when $N = 10^{10}$.

b) Compute $\int_0^1 G(x) \, dx$ if $N = 10^{10}$.

c) Explain b)'s result in relation to a).

4. Find the average value of $f(x) = |x|$ on the interval $[-1, 3]$.

5. Use the information about the function T and its first two derivatives, given in the table to the right, to calculate each of the following definite integrals.

x	$T(x)$	$T'(x)$	$T''(x)$
1	2	-2	2
2	3	6	5
3	7	4	-4

a) $\int_1^3 T'(x) \, dx$ b) $\int_1^3 T''(x) \, dx$ c) $\int_2^3 T(x)T'(x) \, dx$ d) $\int_1^3 xT''(x) \, dx$

6. Calculate each of the indefinite integrals:

a) $\int \sec^4 x \, dx$ b) $\int \sec^3 x \tan x \, dx$ c) $\int \sec^2 x \tan^2 x \, dx$ d) $\int \sec x \tan^3 x \, dx$

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* It isn't always necessary or even possible to compute every integral exactly. But if an estimate can be made . . .