1. Find to four place accuracy the number d so that if the center of two circles of radius 1 are at distance d, the area common to the two circles is half of the area of either circle.

2. a) Compute $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$.

b) Suppose a is a small positive number. Compute $I_a = \displaystyle\int_1^2 \frac{1}{x^2+a}\,dx$.

c) Compute $\displaystyle\lim_{a\to 0+} I_a$ using your result from b).

d) Draw some typical pictures of the area computed by I_a (for small a) and the area computed in a). Do these help to explain any coincidences? Discuss briefly.

3. Many integrals can be done with *rationalizing substitutions* which change the integrals into integrals involving rational functions. These integrals then can be computed using partial fractions. Here are some examples; please compute them.

a) $\displaystyle\int \frac{1}{(\sqrt{x}+1)(\sqrt{x}+2)}\,dx$ (Try $t^2 = x$.)

b) $\displaystyle\int \frac{e^x+1}{e^{2x}+1}\,dx$ (Try $t = e^x$.)

c) $\displaystyle\int \frac{dx}{\sqrt{1+\sqrt{1+x}}}$ (Try $t =$ *something*.)

d) $\displaystyle\int \frac{\cos\theta}{1-(\sin\theta)^2}\,d\theta$ (Try $t = \sin\theta$*.)

4. Compute two definite integrals:

a) $\displaystyle\int_0^{1/3} 4x\sqrt{1-3x}\,dx$

b) $\displaystyle\int_{2\pi}^{3\pi} x \arcsin\left(\frac{\pi}{x}\right)\,dx$

5. Use integration by parts twice to evaluate $\displaystyle\int \sin 7x \cos 5x\,dx$.

---

* You've just integrated sec. I hope you got ln(sec + tan)!