1. Suppose g is defined by $g(x) = 3e^{\cos x}$. `Maple`'s graphs of g and its first four derivatives on the interval $[2, 7]$ are on the back of this page. Suppose I is the value of $\int_2^7 g(x)\,dx$.

a) Use the graph of g to estimate I.

b) Use the information in the graphs to tell how many subdivisions n are needed to use the Trapezoid Rule to estimate I with error $< 10^{-5}$.

c) Use the information in the graphs to tell how many subdivisions n are needed to use Simpson's Rule to estimate I with error $< 10^{-5}$.

2. Assume that a is a positive constant. Suppose R is the region bounded above by $y = \dfrac{1}{x^a}$, below by $y = 0$, and on the left by the line $x = 1$. Determine those values of a for which the integral that results from attempting to calculate each of the following converges:

a) The area of R;

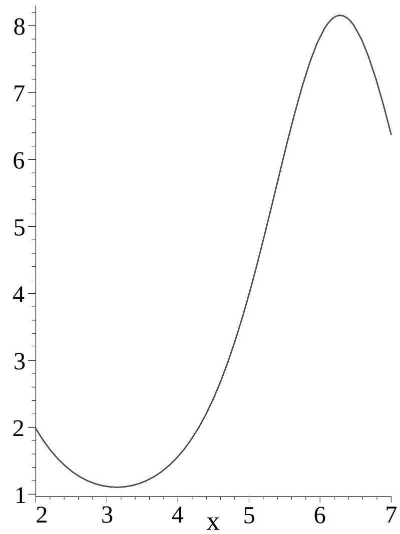
b) The volume of the solid obtained by rotating R around the x-axis;

c) The volume of the solid obtained by rotating R around the y-axis.

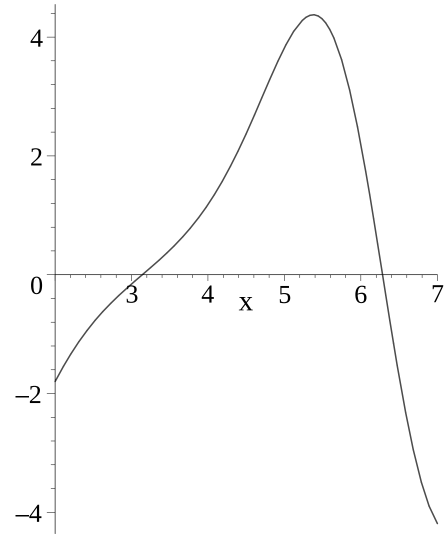
3. Test each of the following integrals for convergence or divergence. Evaluate one of the convergent integrals:

a) $\displaystyle\int_0^\infty \frac{dx}{1+x^4}$ b) $\displaystyle\int_0^\infty \frac{x\,dx}{1+x^4}$ c) $\displaystyle\int_0^\infty \frac{x^2\,dx}{1+x^4}$ d) $\displaystyle\int_0^\infty \frac{x^3\,dx}{1+x^4}$ e) $\displaystyle\int_0^\infty \frac{x^4\,dx}{1+x^4}$

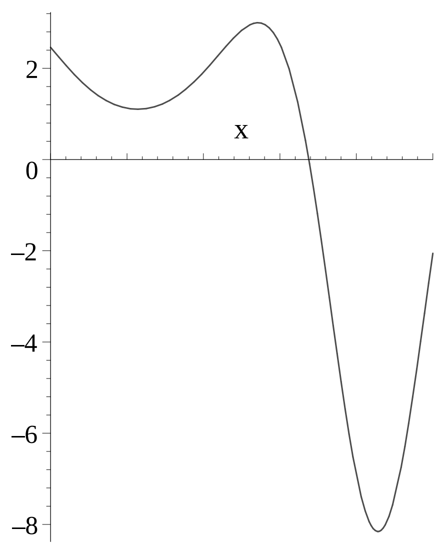
4. In third semester calculus it is proved that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$. Use this to show that $\int_0^\infty x^{-\ln x}\,dx = \pi^{1/2}e^{1/4}$. (`Maple` *can* "do" the first integral, but not the second!)



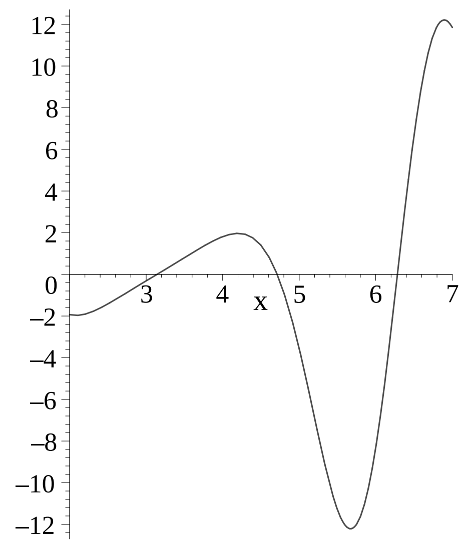
Graph of g



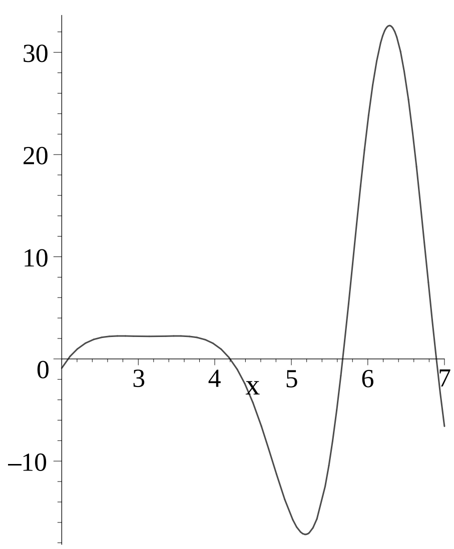
Graph of g'



Graph of g''



Graph of $g^{(3)}$



Graph of $g^{(4)}$