

Below is a real exam (with some slight changes) given last year at this time in Math 152.

1. Find the (finite) area to the right of the y -axis bounded by the line $y = x$ and the curve $y = x^3 - 3x$.

2. A solid has as its base the region to the right of the y -axis bounded by the y -axis and the parabola $x = 4 - y^2$. A cross-section of the solid that is perpendicular to the x -axis is a square. Find the volume of the solid.

3. A (finite) region R in the first quadrant is bounded by the curves: $y = x^{1/2}$ and $y = x^3$. Find the volume of the solid that results when R is rotated:

a) About the x -axis; b) About the y -axis.

4. Find the average value of the function $f(x) = (\sin x)^3$ on the interval $[0, \pi]$. Show how the integral is calculated.

5. Calculate the following integral giving an exact answer in terms of mathematical constants such as π and e , not a numerical approximation: $\int_1^2 x^2 \ln x \, dx$

6. Calculate the following integrals, showing your work:

a) $\int \frac{x+1}{x^3+x} \, dx$ b) $\int \frac{x^2 \, dx}{\sqrt{1-x^2}}$ c) $\int \frac{dx}{e^x+1}$

7. Determine which of the following integrals converge and which diverge, giving your reasons and the value of those that are convergent.

a) $\int_0^\infty x e^{-x^2} \, dx$ b) $\int_2^\infty \frac{dx}{x \ln x}$

8. The integral $\int_0^1 \cos(x^3) \, dx$ is approximated using the Trapezoidal Rule by dividing $[0, 1]$ into n segments of equal length. How large should n be in order to guarantee that the error is at most 10^{-6} ?

9. The region R to the right of the line $x = 1$, bounded by the curve $y = \frac{1}{x^2}$ and the x -axis, is infinite in extent but, nevertheless, has finite area. Find a number c so that the vertical line $x = c$ divides the region R into two subregions of equal area.

Note About problem 8: please write (but do *not* compute) the sum corresponding to the Trapezoid Rule approximation when $n = 4$.

Office hours I'll be in Hill 542 on Wednesday, October 10, from 4:30 PM to about 6 PM, available to answer questions and otherwise help you prepare for the exam.

An excursion to meet Maple I've reserved an "Instructional Microcomputing Lab", ARC 118, for Friday, October 12, from 11:30 AM to 12:50 PM (all of 3rd period). You should have an Eden account for this session which will introduce some of Maple's capabilities. I'll *try* to have your exams, graded, to return to you at the end of this period.