Differentiate:

1. $\dfrac{x^2+3}{5-9x^4}$

2. $\sin\left(\dfrac{1}{x^3}\right)$

3.. $\sqrt{2x^5+9x}$

4. $\ln(3+4e^x)$

5.. $\arctan(3e^{5x})$

6. $x^2\cos(1+2x)$

7. $(1+x^n)^{1/n}$ (Here "n" is supposed to be a constant.)

8. $\arcsin(e^x)-e^{\arcsin(x)}$

9. $\big(\cos(x^5)\big)^7$

10. $\sqrt{1+\sqrt{2+\sqrt{3+x^5}}}$

11. $3+\sec\left(\dfrac{5+2x}{x^2}\right)$

12. $\tan(-5x^3)$

Here is information about the functions f and g and their derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	3	1	4
0	1	3	2	3
1	5	-2	2	-3
2	1	0	3	1
3	2	5	3	2

1. If $F(x) = f(g(x))$, compute $F(2)$ and $F'(2)$.

2. If $G(x) = g(x^2)$, compute $G(-1)$ and $G'(-1)$.

3. If $H(x) = g(x)^{-5}$, compute $H(0)$ and $H'(0)$.

4. If $K(x) = \dfrac{f(x)}{g(3x-1)}$, compute $K(1)$ and $K'(1)$.

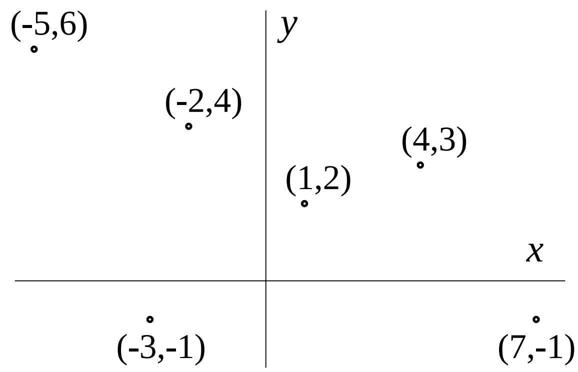
Suppose $r(x) = (5 + x^2)^{5/2}$.

5. Compute $r(2)$ and $r'(2)$.

6. Use a linear approximation to estimate $r(2.03)$ and $r(1.98)$. (You need not carry out the arithmetic!)

7. Are these estimates likely to be larger or smaller than the true values? (Hint: compute $r''(2)$ [or at least the sign of $r''(2)$!]).

Here are some points on the graph of a function, W.



1. Is W' ever negative? Is W' ever positive?

2. Find a specific negative number, n, so that $W'(x) = n$. Where is your x located, as precisely as possible? Find a specific positive number, p, so that $W'(x) = p$. Where is your x located, as precisely as possible?

3. Repeat question 1, with W'' in place of W'. Also explain why.

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4. Find or describe as precisely as you can, the 5$^{\text{th}}$ derivative of

a) $A(x) = \cos(2x)$ b) $B(x) = x^{12} - 7x^2$ c) $C(x) = \sqrt{1+x}$.

5. Find or describe as precisely as you can, the 105$^{\text{th}}$ derivative of

a) $A(x) = \cos(2x)$ b) $B(x) = x^{12} - 7x^2$ c) $C(x) = \sqrt{1+x}$.

Problems 1–12 can be checked with `Maple`, although sometimes the answers are written in a form different from what you might expect. For example, the answer `Maple` gives for problem 10 is

$$\frac{5}{8}\frac{x^4}{\sqrt{1+\sqrt{2+\sqrt{3+x^5}}}\sqrt{2+\sqrt{3+x^5}}\sqrt{3+x^5}}$$

while "by hand" my answer is

$$\frac{1}{2}\left(1+\sqrt{2+\sqrt{3+x^5}}\right)^{-1/2}\frac{1}{2}\left(2+\sqrt{3+x^5}\right)^{-1/2}\frac{1}{2}\left(3+x^5\right)^{-1/2}\left(5x^4\right)$$

and these are certainly the same but they initially look a bit different.

For problems 1–5, you must use the Chain Rule (and other differentiation algorithms) carefully. The numerical answers follow:

1. 2 and 5.

2. 2 and 6

3. $\frac{1}{32}$ and $-\frac{15}{64}$.

4. $-\frac{2}{3}$ and $-\frac{21}{9}$.

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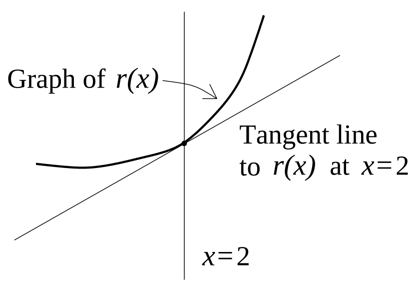
5. $r(2) = 243$ and $r'(2) = 270$.

6. Using $r(x+h) \approx r(x) + r'(x)h$ with $h = +.03$ first, we get $r(2.03) \approx 243 + (.03)(270)$. If h is then $-.02$, $r(1.98) \approx 243 - (.02)(270)$.

7. Since $r''(x) = 5(5x^2)^{3/2} + 5x\left(\frac{3}{2}\right)(5+x^2)2x$, plugging in $x = 2$ shows that $r''(2) > 0$.



That means $r(x)$ is concave up near 2. Therefore the estimates above are below the true values.

1. Yes and yes.

2. The MVT (Mean Value Theorem) predicts that there is an x_1 between -5 and -3 with $W'(x_1) = \frac{6-(-1)}{-5-(-3)} = -\frac{7}{2}$ (so this is an n). Similarly, there is x_2 between -3 and -2 with $W'(x_2) = \frac{4-(-1)}{-2-(-3)} = 5$ (so this is a p).

3. Yes and yes. Since $x_1 < -3 < x_2$ and $W'(x_1) = -\frac{7}{2} < 3 = W'(x_2)$ by applying the MVT to W', there is x_3 between x_1 and x_2 so that $W''(x_3) = \frac{W(x_2)-W(x_1)}{x_2-x_1}$. But $x_2 - x_1 > 0$ and $W(x_2) - W(x_1) = 3 - -\frac{7}{2} = \frac{13}{2}$. So $W''(x_3) = \frac{\frac{13}{2}}{\text{positive number}}$, which is positive. You can get x_4 with $W''(x_4)$ <u>negative</u> by using $(-3,-1)$, $(-2,4)$, and $(1,2)$.

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4. a) $A^{(5)}(x) = 2^5\left(-\sin(2x)\right)$.

b) $B^{(5)}(x) = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8x^7$.

c) $C^{(5)}(x) = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)(1+x)^{-9/2}$

5. a) $A^{(105)}(x) = 2^{105}\left(-\sin(2x)\right)$.

b) $B^{(105)}(x) = 0$.

c) $C^{(105)}(x) = \frac{\text{(Product of all odd integers from 1 to 207)}}{2^{105}}(1+x)^{-209/2}$.