(14) 1. Use the method of partial fractions to verify that

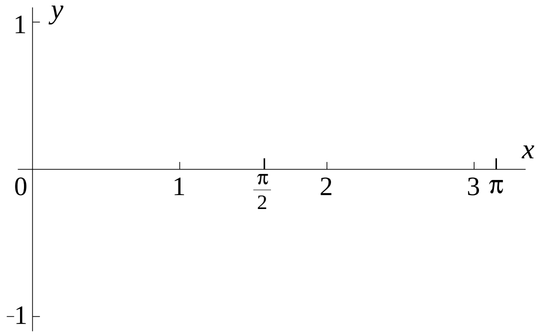
$$\int_0^1 \frac{1}{(x+1)(x^2+1)}\,dx = \frac{1}{4}\ln 2 + \frac{1}{8}\pi\,.$$

(8) 2. Suppose $\mathcal{R}$ is the region bounded by $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$.

a) Find the volume of the solid that results from rotating $\mathcal{R}$ around the x-axis.

b) Find the volume of the solid that results from rotating $\mathcal{R}$ around the y-axis.

(12) 3. a) Carefully sketch $y = \sin x$ and $y = \sin 2x$ on the axes given for $0 \leq x \leq \pi$. Be sure to label each of the curves.



b) Find **all** points of intersection of the curves sketched in part a).

c) Find the area enclosed between the curves sketched in part a). Give an exact answer in terms of mathematical constants such as π and e.

(10) 4. a) Suppose w is a positive number. Define $A(w)$ to be the average value of $(\cos x)^2$ on the interval $0 \leq x \leq w$. Compute $A(w)$, and show how the integral is calculated.

b) What is the limit of $A(w)$ as $w \to 0^+$?

(12) 5. a) Explain why $\displaystyle\int_1^\infty \frac{1}{x^2+e^{2x}}\,dx$ converges.

b) Explain why the value of the integral in a) is less than $\frac{1}{2}$.

(12) 6. Calculate the following integrals, showing your work.

a) $\displaystyle\int \frac{dx}{\sqrt{1+x^2}}$

b) $\displaystyle\int \frac{dx}{2e^x+1}$

(10) 7. Use a substitution followed by integration by parts to verify that

$$\int_0^1 e^{\sqrt{x}}\, dx = 2\,.$$

(12) 8. a) Suppose m and n are positive integers. Find a reduction formula for

$$\int x^m (\ln x)^n \, dx \, .$$

(Here the aim is to have an equation with the integral given here on one side and a similar integral with reduced n on the other side, since if we can push n to 0 we'll just have a polynomial to integrate, which is easy.)

b) Use the formula obtained in a) to compute

$$\int x^{20} (\ln x)^2 \, dx \, .$$

(Do not "simplify" your answer, please!)

(10) 9. The integral $\int_0^2 \left(x^3+1\right)^{7/2}\, dx$ is approximated using the Trapezoidal Rule by dividing $[0,2]$ into n segments of equal length. How large should n be in order to guarantee that the error is at most 10^{-6}?

Note You must give some reason explaining why any overestimates of derivatives you make are valid on the entire interval.

First Exam for Math 152, section 72

October 11, 2001

NAME ______________________________

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No student notes and no calculators with graphing or symbolic capability may be used on this exam.

A formula sheet will be handed out with the exam.

Problem Number	Possible Points	Points Earned:
1	14	
2	8	
3	12	
4	10	
5	12	
6	12	
7	10	
8	12	
9	10	
Total Points Earned:		