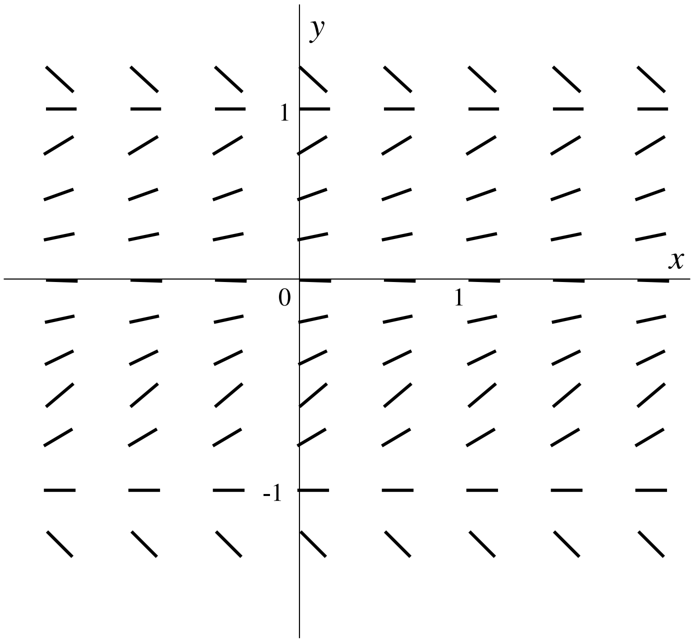
(8) 1. Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{xy^3}{x^2+1}$$

satisfying the initial condition $y(0) = 3$. In the answer express y explicitly as a function of x.

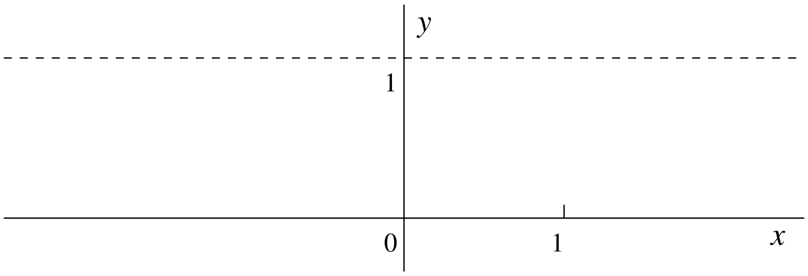
(12) 2. Below is part of the direction field for the differential equation $y' = y^2(1-y)(1+y)$.



a) List all numbers k so that the constant function $f(x) = k$ is a solution of this differential equation (these are the *equilibrium solutions*).

$k =$ ____________

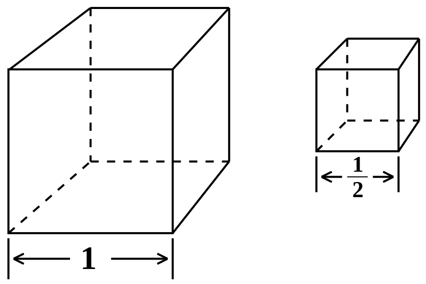
b) Sketch a typical solution curve $y = f(x)$ to this differential equation when $0 < y(0) < 1$.



What is $\lim_{x\to+\infty} f(x)$? ____________

What is $\lim_{x\to-\infty} f(x)$? ____________

(12) 3. A sequence of cubes has edges made of thin wire. The largest cube has edge length 1 inch, and each successive cube has edge length half the size of the preceding one. The first two cubes are shown here.



a) What is the total length of wire needed to construct the edges on *all* of the cubes?

b) What is the total volume enclosed by *all* of the cubes?

(10) 4. Find the interval of convergence and the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{\sqrt{n}}.$$

In addition, determine whether the series is absolutely or conditionally convergent at the boundary points of the interval of convergence.

The radius of convergence is ____________ .

The interval of convergence is ____________ .

(10) 5. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^2+5n+7}$ converges. Find a specific finite sum of rational numbers (quotients of integers) which is within .0001 of the sum of the infinite series. Be sure to explain why your error estimate is correct.

Hint Compare the "infinite tail" to something simpler, and analyze that.

(10) 6. a) Suppose the sequence $\{A_n\}$ is defined by $A_n = \dfrac{6 \cdot 4^n + 7n^3}{5 \cdot 4^n + 8n^2}$. What is $\lim_{n\to\infty} A_n$?

b) Suppose the sequence $\{B_n\}$ is defined by $B_n = \left(1 + \dfrac{3}{n}\right)^{5n}$. What is $\lim_{n\to\infty} B_n$?

Hint ln and l'H.

(10) 7. Find a specific polynomial $p(x)$ so that $|p(x) - x^{1/4}| < .001$ for $16 \leq x \leq 17$. Be sure to explain why your error estimate is correct.

n	2^n
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
11	2,048
12	4,096
13	8,192

(12) 8. a) Use the Taylor series for the exponential function to write $\int_0^1 e^{-x^2}\,dx$ as an infinite series. Use summation notation in your answer.

b) Find a specific finite sum of rational numbers (quotients of integers) which is within .00001 of the true value of $\int_0^1 e^{-x^2}\,dx$. Be sure to explain why your error estimate is correct.

n	$n!$
1	1
2	2
3	6
4	24
5	120
6	720
7	5,040
8	40,320
9	362,880
10	3,628,800

(10) 9. Estimate the maximum error committed when $\cos x$ is replaced by $1 - \dfrac{x^2}{2} + \dfrac{x^4}{24} - \dfrac{x^6}{720}$ for x in the interval $[-2, 2]$. Be sure to explain why your error estimate is correct.

(6) 10. a) What is the 300^{th} Taylor polynomial for $f(x) = 3 - 11x^5$ centered at $a = 0$? Why?

b) Explain briefly why $g(x) = |x|$ has no Taylor series centered at $a = 0$.

Second Exam for Math 152, section 72

November 29, 2001

NAME ______________________________

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No student notes and no calculators may be used on this exam. A formula sheet will be handed out with the exam.

Problem Number	Possible Points	Points Earned:
1	8	
2	12	
3	12	
4	10	
5	10	
6	10	
7	10	
8	12	
9	10	
10	6	
Total Points Earned:		