640:152:72 **Part III: playing with calculus on `Maple`** **10/12/2001**

The basic calculus commands do differentiation and integration. Let's try them immediately.

*diff(3*x^7 - 22.1*x^2,x);* **RET**

and

*int(x*sqrt(x+2),x);* **RET**

Please realize that this integration is not too easy: `Maple` either substituted or it integrated by parts (it can do both).

Probably `Maple` knows all the functions you do, and a number of others. The function $\sin\left(x^2\sqrt{x+1}\right)$ and the appropriate calculus rules are known. Let's first assign this expression a name and then play.

*Q:=sin(x^2 * sqrt(x+1));* **RET**

diff(Q,x); **RET**

This should get the first derivative. How many ways can you think of to get the second derivative? Here are a few. First, immediately after typing the command above and getting a response, type

diff(% ,x); **RET**

This will do it. For an independent computation, try the command line

diff(Q,x,x); **RET**

How about the tenth derivative? First type

x$10; **RET**

to see what the symbol *$* does. Please now find the 10^{th} derivative of the function $\sin\left(x^2\sqrt{x+1}\right)$ with very little typing. You may now realize (even if you <u>want</u> to compute the 10^{th} derivative of this function) why people end commands with *:* (which turns off the output) rather than with *;* which displays the output. You may want the results for some purpose, but you may not have the need or desire to actually look at it!

What is the coefficient of x^3 in the seventh derivative of $\left(x^2+\frac{1}{x^2}\right)^5$? First compute the indicated derivative. You'll get a mess. Then have `Maple` massage the result algebraically so you can read off the answer. I'm an amateur and first tried *expand(%)* and I also tried *simplify(%)* and the results were different. You can also try it by first asking `Maple` to expand $\left(x^2+\frac{1}{x^2}\right)^5$ and then differentiate the result seven times. I hope the answer will be the same.

Let's look at integration a bit more closely. Define V to be $e^{\sin x}$:

V:=exp(sin(x)); **RET**

Now let's integrate it. First (read and try this carefully!) type

int(V,w); **RET**

and explain the result to yourself. Remember, a program will do what you tell it to do! Now try

int(V,x); **RET**

and you may need to wait a bit and then have something else to explain. `Maple` knows the usual integration algorithms and many, many other antidifferentiation tricks. An answer like this is a fairly good hint that it "can't be done": that is, the antiderivative can't be expressed in terms of familiar functions with familiar ways of combining them, including sum, product, composition, ... even using the rather large collection of functions `Maple` knows.

We can also compute definite integrals. For example,

int(x^3,x=1/7..b); **RET**

computes $\int_{\frac{1}{7}}^{b} x^3dx$ (if you want it!). `Maple` indicates a range (for integration and for other purposes) by the notation *variable=lower limit..upper limit*.

Remember V? Let's be sure `Maple` does (check by typing *V;*) and then compute $\int_0^1 e^{\sin x}\,dx$ by typing

int(V,x=0..1); **RET**

and consider the result. Perhaps we are disappointed, but I can be happier if I follow that answer with

evalf(%); **RET**

OVER

You can evaluate V itself with a command like

subs({x=3}, V); **RET**

followed by

evalf(%); **RET**

If you're not scared, you could have done this all together by typing

evalf(subs({x=3}, V)); **RET**

but sometimes I get confused by the matching required (in count and type) of all the parentheses. We could similarly evaluate a derivative of V by differentiating with *diff*, *subs*tituting, and then *evalf*uating. Or we could define our own functions. Initially the syntax may seem burdensome, but let's try it with a simple example.

N:=x->arctan(x^3); **RET**

This means: let's call N the function which assigns $\arctan(x^3)$ to the input x. The "arrow" is gotten by typing a dash followed by the "less than" symbol. Type

N(2); **RET**

and

*N(2*z);* **RET**

to make sure that `Maple` understands that N is a function. Now type

diff(N(x),x); **RET**

That's now an **expression** and not a function. There's no way to "plug in" another expression like $K+3$ in it easily (yes, we could use the *subs* command, but that's cumbersome). The designers of `Maple` have another way to differentiate functions (such as N) rather than expressions (such as Q). Try

D(N); **RET**

and view the result. Indeed: call it by a new name, say, M:

M:=D(N); **RET**

Now evaluate $M(3)$ and $M(K+3)$.

Please check out the difference between

int(N(x),x); **RET**

and

int(N,x); **RET**

I find one of these answers amusing to look at (because I didn't have to compute it and I can appreciate the work involved!) and the other seems almost silly: the function hasn't been told what to evaluate, so `Maple` can't integrate.

The difference between "expressions" like V and "functions" like N can be subtle. Expressions seem more static, while functions have a more dynamic aspect – substituting is built into their structure.

`Maple` also knows about such calculus topics as limits, sums, and series. I suggest that when you need to work with any of these, at that time please try *help(limit)* and *help(sum)* and *help(series)*. I almost always look at the examples first. Don't be afraid to try some experiments, and look at the SEE ALSO pages if the command you are investigating doesn't do exactly what you want.

It's time to graph.

Disclaimer! Non-advertisement!! Important information!!!

Symbolic manipulation programs such as `Maple` are becoming increasingly available. Other popular programs with about the same capabilities are `Derive` and `Mathematica` and there are many special purpose programs in various fields of science, engineering, and mathematics which have extensive "intelligence" to analyze models. We're considering `Maple` here because Rutgers has a site license for this program, and it should be generally available on Rutgers systems. The specific instructions won't be the same from program to program, but many of the same ideas will be present. Students should expect to have a machine do tiresome or elaborate symbolic computations as well as numerical computations.