1. Recently Andrew Wiles proved that there were no solutions to the Fermat equation

$$a^n + b^n = c^n$$

if a, b, c, and n are positive integers with $n > 2$. (There are, of course, solutions when $n = 2$: for example, $3^2 + 4^2 = 5^2$.)

a) Does the equation

$$4^x + 5^x = 6^x$$

have any solution?* If it does, find an approximate solution with accuracy $\pm .001$. If it does not, explain why.

b) Suppose a, b, and c are positive real numbers. Explore whether the equation

$$a^x + b^x = c^x$$

must have a solution.

Comment You are not asked to provide a formula for x. You are asked to find conditions which will guarantee that such an x either does or does not exist.

3. Suppose you know that $H'(x) = \dfrac{2}{1+x^4} - \dfrac{3}{4+x^4}$. Is $H(0) < H(1)$?

Note It is not likely now that you can write a formula for a function with this derivative (and, by the way, such a formula wouldn't really help very much!). So you must use the information you have about the derivative.

3. Consider the function

$$F(x) = \frac{e^{Ax}}{1+e^{Ax}}$$

where A is a constant.

a) Make a rough sketch of $y = F(x)$ for x in $[-1, 1]$ if $A = 10^{10}$.

b) Make a rough sketch of $y = F(x)$ for x in $[-1, 1]$ if $A = 10^{-10}$.

c) Make a rough sketch of $y = F(x)$ for x in $[-1, 1]$ if $A = -10^{10}$.

d) Make a rough sketch of $y = F(x)$ for x in $[-1, 1]$ if $A = -10^{-10}$.

4. Suppose that Q is the function

$$Q(x) = \Big(\arctan\big(\ln(\sqrt{x} - 1)\big)\Big)^3 .$$

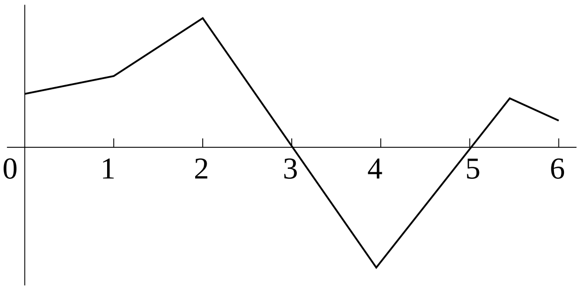
a) What are the domain and range of Q? Answers should *not* be numerical approximations, but should be written if needed in terms of constants studied in calculus such as π and e.

b) Suppose $y = Q(x)$. Write a formula for x in terms of y.

OVER

* The word "integer" does not appear in this sentence!

5. The domain of the function g is all numbers between 0 and 6. The graph of g is below.



The graph of g

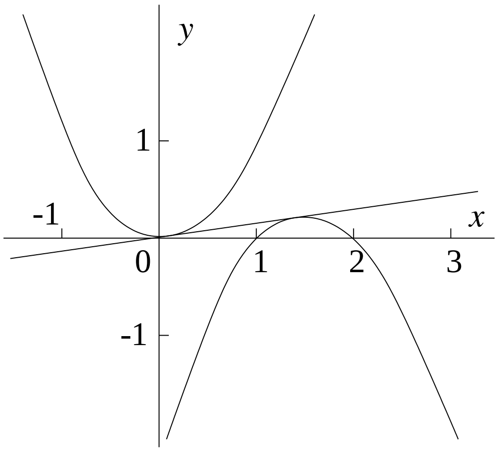
Suppose $f(x) = \int_0^x g(t)\,dt$, so that f also has domain $0 \le x \le 6$. Answer the following questions as well as you can based on the information you have.

a) Where is f increasing? Where is f decreasing?

b) Does f have any local extrema (max or min)? Identify them.

c) Where is f concave up? Where is f concave down?

6. Consider the parabola $y = x^2$. Flip it and move it right, to create a parabola opening "down" and intersecting the $x-$axis at $x = 1$ and $x = 2$. An equation for such a parabola is $y = -(x-1)(x-2)$. Look below: there is a straight line tangent to both parabolas.



Find an equation for the line.

An example of a workshop problem writeup is on the web page
`http://www.math.rutgers.edu/courses/151-152/old/writeup.html`
with accompanying discussion. The writeup is a bit too elaborate, but the principles displayed there are good. In your writeup, include any information (such as pictures and computations) that you think is useful. Label any pictures.

Your written solution should be of high-quality, with the explanation given in complete sentences. You will be graded both on mathematical content and on presentation. Neatness counts. While I encourage you to discuss the problem with other students and with me, the written work you hand in must be your own.