(18) 1. a) Find an equation for the plane tangent to the surface $x^2yz = 6$ at the point $(1,2,3)$.

b) A particle has position vector given by $\mathbf{R}(t) = \cos(2t)\mathbf{i} + \sin(t^2)\mathbf{j} - 3t\mathbf{k}$. Find parametric equations for a line tangent to the path of this particle when $t = 0$.

c) The plane found in a) and the line found in b) intersect. Find the point of intersection.

(12) 2. Suppose $\begin{cases} x = u + v^2 \\ y = 2uv - v^3 \end{cases}$. Note that when $u = 3$ and $v = 1$, then $x = 4$ and $y = 5$.

a) Suppose u is changed to 3.02 and v is changed to .96. Use linear approximations to estimate the "new" values of x and y.

b) Suppose that we wish to *estimate* what values of u and v near $u = 3$ and $v = 1$ will give the values $x = 4.07$ and $y = 5.03$. Use linear approximations *backwards* to estimate such values.

(10) 3. Suppose $f(x,y) = x^2y$. Then $f(2,3) = 12$.
There is $H > 0$ so that if $\|(x,y) - (2,3)\| < H$ then $|f(x,y) - f(2,3)| < \dfrac{1}{1{,}000}$.
Find such an H and explain *why* your assertion is correct.

(12) 4. If $x = s^2 - t^2$, $y = 2st$, and $z = F(x,y)$, show that $\frac{\partial^2 z}{\partial s^2} + \frac{\partial^2 z}{\partial t^2} = 4\sqrt{x^2+y^2}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$.

(12) 5. Suppose $F(a,b,c) = a^2b + \sqrt{bc + 2c}$.

a) Compute $\dfrac{\partial F}{\partial a}$, $\dfrac{\partial F}{\partial b}$, and $\dfrac{\partial F}{\partial c}$ at the point p with coordinates $a = 2$, $b = 1$, and $c = 3$.

b) In what direction will F increase most rapidly at p? Write a unit vector in that direction. You do **not** need to "simplify" your answer!

c) What is the directional derivative of F at p in the direction found in b)? You do **not** need to "simplify" your answer!

(12) 6. Leonhard Euler (1707–1783) was a great and very prolific mathematician. He published *Institutiones Calculi Differentialis* (*Methods of the Differential Calculus*) in 1755. It was an influential text, and was the first source of criteria for discovering local extrema of functions of several variables. In it Euler investigated the following specific example: $V = x^3 + y^2 - 3xy + \frac{3}{2}x$. He asserted that V has a minimum both at $x = 1$ and $y = \frac{3}{2}$ and at $x = \frac{1}{2}$ and $y = \frac{3}{4}$. Was Euler correct?

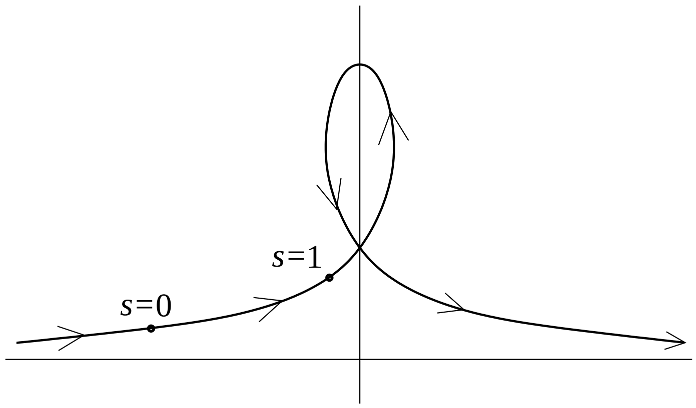
Reference: *A History of Mathematics* by Victor J. Katz, Harper Collins, 1993, p. 517.

You need to check that Euler found all the c.p.'s and only the c.p.'s, and also that he classified them correctly.

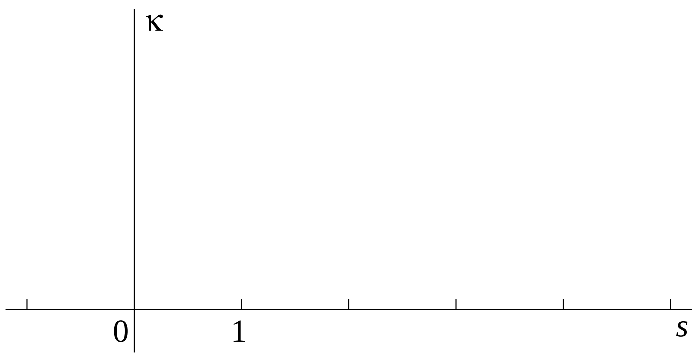
(10) 7. A particle has position vector given by $\mathbf{R}(t) = \frac{1}{t}\mathbf{i} + t^2\mathbf{j} - 3t\mathbf{k}$. a) What are the velocity and acceleration vectors of this particle when $t = 1$?

b) Write the acceleration vector when $t = 1$ as a sum of two vectors, one parallel to the velocity vector when $t = 1$ and one perpendicular to the velocity vector when $t = 1$.

(8) 8. The curve below is parameterized by arc length, s. Arc length is measured forward and backward from the indicated initial point where $s = 0$.



Sketch a graph of the curvature, $\kappa(s)$, of this curve as well as you can on the axes below.



(6) 9. Explain briefly why the following limit does not exist. $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

First Exam for Math 291, section 1

October 16, 2002

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
You may use a calculator only during the last 20 minutes.
No notes may be used on this exam. The last page contains formulas.

Problem Number	Possible Points	Points Earned:
1	18	
2	12	
3	10	
4	12	
5	12	
6	12	
7	10	
8	8	
9	6	
Total Points Earned:		