(14) 1. a) Compute $\int_2^3 \int_{1/x}^{x^2} x^2 y \, dy \, dx$.

b) Write this iterated integral in $dx\,dy$ order. You may want to begin by sketching the area over which the double integral is evaluated. You are **not** asked to evaluate the $dx\,dy$ result, which may be one or more iterated integrals.

(12) 2. Suppose $Q$ is the collection of points in the $xy$-plane which are both inside the first quadrant and outside the unit circle. Compute $\iint_Q \frac{1}{(x^2+y^2+1)^3}\, dA$.

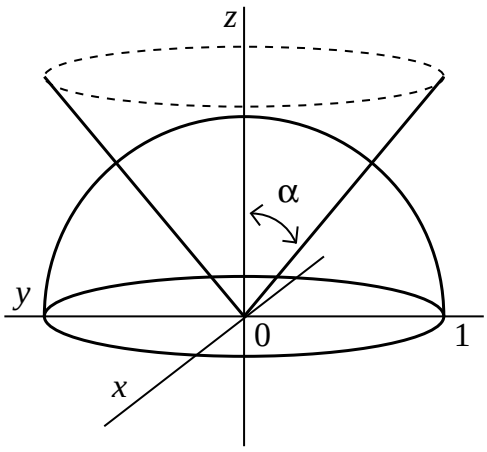
(10) 3. Compute the volume of the solid bounded by the $xz$-plane, the $yz$-plane, the $xy$-plane, the planes $x = 1$ and $y = 1$, and the surface $z = x^2 + y^4$.

(16) 4. Find the maximum and minimum values of $F(x, y, z, w) = x + 2y + 3zw$ for points $(x, y, z, w)$ in $\mathbb{R}^4$ satisfying $x^2 + y^2 + z^2 + w^2 = 1$.

(16) 5. Integrate the function $y$ over the region in the first quadrant of the $xy$-plane bounded by the curves $y = \frac{1}{x}$ and $y = \frac{4}{x}$ and $y = 9x$ and $y = 16x$ **using the change of variables technique**. I suggest you try the variables $s = xy$ and $t = \frac{y}{x}$. Describe the corresponding region in the $st$-plane, compute the area distortion factor (Jacobian), and rewrite the double integral as a $ds\,dt$ integral.

**Comment** The function, the Jacobian, and the region *should* interact to produce a result easy to deal with, since this is an invented example. The answer is $\frac{14}{3}$.

(12) 6. In this problem $H$ is the upper half of the unit sphere in $\mathbb{R}^3$: those $(x, y, z)$ with $x^2+y^2+z^2 \le 1$ and $z \ge 0$. There is a right circular cone whose vertex is $(0, 0, 0)$ and whose axis of symmetry is the positive $z$-axis which divides the volume of $H$ into two equal parts. Find the angle $\alpha$ that determines this cone. The diagram defines $\alpha$, which is the angle that the positive $z$-axis makes with a line on the cone through the vertex.




(20) 7. a) Suppose $C$ is the boundary of the unit circle oriented in the usual (counterclockwise) fashion. Compute $\int_C \left(y^2 + \sqrt{1 + \cosh(\cos x)}\right) dx + (x + e^{\arctan y})\, dy$.

b) Suppose $D$ is the path consisting of three straight line segments, first from $(1, 2)$ to $(4, -3)$, then from $(4, -3)$ to $(2, 6)$, and then from $(2, 6)$ to $(3, 4)$. Compute $\int_D (2xy^3)\, dx + (3x^2y^2 + 4y^3)\, dy$.

# Second Exam for Math 291, section 1

November , 2002

**NAME** _____

**Do all problems, in any order.**
**Show your work. An answer alone may not receive full credit.**
**You may use a calculator only during the last 20 minutes.**
**No notes may be used on this exam. A page with formulas will be supplied.**

| Problem Number | Possible Points | Points Earned: |
|---|---|---|
| 1 | 14 | |
| 2 | 12 | |
| 3 | 10 | |
| 4 | 16 | |
| 5 | 16 | |
| 6 | 12 | |
| 7 | 20 | |
| Total Points Earned: | | |