Formula sheet for the second exam in Math 291, fall 2002

FIRST VERSION 11/25/2002

Lagrange multipliers for one constraint

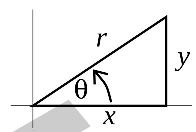
If G(the variables) = a constant is the constraint and we want to extremize the objective function, F(the variables), then the extreme values can be found among F's values of the solutions of the system of equations $\nabla G = \lambda \nabla F$ (a vector abbreviation for the equations $\lambda \frac{\partial F}{\partial \star} = \frac{\partial G}{\partial \star}$ where $\star$ is each of the variables) **and** the constraint equation.

Polar coordinates

$x = r\cos\theta \quad y = r\sin\theta$

$r^2 = x^2 + y^2 \quad \theta = \arctan\left(\frac{y}{x}\right)$

$dA = r\ dr\ d\theta$

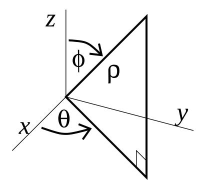


Spherical coordinates

$x = \rho\,\sin\phi\,\cos\theta \;\; y = \rho\,\sin\phi\,\sin\theta \;\; z = \rho\,\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

$dV = \rho^2 \sin\phi\; d\rho\; d\theta\; d\phi$



Change of variables in 2 dimensions

$\int\int_R f(x,y)\,dA = \int\int_{\tilde{R}} f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\ dv$

where $\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$ is called the Jacobian.

If $\rho(x,y,z)$ is density in a region R of $\mathbb{R}^3$, the total mass in R is $\int\int\int_R \rho(x,y,z)\,dV$.

Green's Theorem

$\int_C P\,dx + Q\,dy = \int\int_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

These P's and Q's give R's area:

$P{=}-y$ and $Q{=}0$

$P{=}0$ and $Q{=}x$

$P{=}-\frac{1}{2}y$ and $Q{=}\frac{1}{2}x$