Formula sheet for the second exam in Math 291, fall 2002

FIRST VERSION 11/25/2002; REVISED 11/26/2002

Lagrange multipliers for one constraint

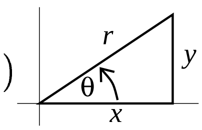
If G(the variables) = a constant is the constraint and we want to extremize the objective function, F(the variables), then the extreme values can be found among F's values of the solutions of the system of equations $\nabla G = \lambda \nabla F$ (a vector abbreviation for the equations $\lambda \frac{\partial F}{\partial \star} = \frac{\partial G}{\partial \star}$ where $\star$ is each of the variables) **and** the constraint equation.

Polar coordinates

$x = r\cos\theta \quad y = r\sin\theta$

$r^2 = x^2 + y^2 \quad \theta = \arctan\left(\frac{y}{x}\right)$

$dA = r\,dr\,d\theta$

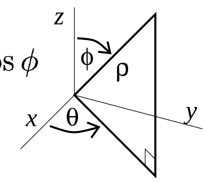


Spherical coordinates

$x = \rho\,\sin\phi\,\cos\theta \quad y = \rho\,\sin\phi\,\sin\theta \quad z = \rho\,\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

$dV = \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$



Change of variables in 2 dimensions

$\int\int_R f(x,y)\,dA = \int\int_{\tilde{R}} f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$; $\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$, the Jacobian.

Total mass of a mass distribution $\rho(x,y,z)$ over a region R of $\mathbb{R}^3$ is $\int\int\int_R \rho(x,y,z)\,dV$.

Line integral formulas

$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot \mathbf{r}'(t)\,dt = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$

$\int_C P(x,y)\,dx + Q(x,y)\,dy = \int_a^b P(x(t),y(t))x'(t)\,dt + Q(x(t),y(t))y'(t)\,dt$

Green's Theorem

$\int_C P\,dx + Q\,dy = \int\int_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

These P, Q pairs will give R's area $\begin{cases} P = -y \text{ and } Q = 0 \\ P = 0 \text{ and } Q = x \\ P = -\frac{1}{2}y \text{ and } Q = \frac{1}{2}x \end{cases}$

A **conservative vector field** $\mathbf{V} = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is a **gradient vector field**: there's $f(x,y)$ with $\nabla f = \mathbf{V}$ so $\frac{\partial f}{\partial x} = P$ and $\frac{\partial f}{\partial y} = Q$. f is a **potential** for $\mathbf{V}$. A conservative vector field is **path independent**. Work done by such a vector field over a **closed curve** is 0. For V conservative with potential f: $\int_C P\,dx + Q\,dy = f(\text{THE END}) - f(\text{THE START})$.

If $P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is conservative, then $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$. If the region is **simply connected** (means **no holes**) then the converse is true, and f is both $\int P(x,y)\,dx$ and $\int Q(x,y)\,dy$.