Useful information for the first exam in Math 291, Fall 2002

The time, date, and place will be:

Hill Center 425, Wednesday, October 16, at 4:30 PM

The exam will cover what we have done up to now, including material up to and including section 14.7 of the text. Note that some of the emphasis of the class meetings has been different from what's in the text. I have reserved the room above for periods 6 and 7.

Some rules for the exam:

• No books or notes.
I'll write a formula sheet you can use. Please *send me formulas* you'd like to have on or before Monday, October 14. I'll write a draft for comments and put it on the web. You'll get copies of the sheet with the exam.

• You may use a calculator only during the last 20 minutes of the exam.
Please leave answers in "unsimplified" form – so $15^2+(.07)\cdot(93.7)$ is preferred to 231.559. You should know simple exact values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$. Traditional math constants such as π and e should be left "as is" and not approximated.

• Show your work: an answer alone may not receive full credit.

Here are some problems from past exams. There are almost three times as many problems as a "real" exam would have.

I'll have a **review session** on **Tuesday, October 15, in Hill 525 at 7:30 PM** to discuss these problems and other questions. I hope that students will *e-mail me the numbers of textbook problems in chapter 14* to go over during Monday's class. Also each student named below should send me a solution of that student's problem for posting on the web.

1. ELLWAY Sketch two level curves of $f(x,y)=\frac{x-y}{x+y}$. Explain why $\lim_{(x,y)\to(0,0)}\frac{x-y}{x+y}$ does or does not exist.

2. GRADZIADIO Let $f(x,y)=xy^2+2x^3$.
a) If one stands on the graph of f at (1,2,6), what is the slope of the graph at that point in its steepest direction?
b) Find $D_{\mathbf{u}}f(1,2)$ if $\mathbf{u}$ is the unit vector in the direction from $(1,2)$ to $(2,4)$.

3. GRUBIN Find an equation of the plane containing the points $(1,2,0)$ and $(0,2,1)$ and parallel to the line $x=1+t$, $y=-1+t$, $z=2t$.

4. GURKOVICH If $z=f(x,y)$, $x=3uv$, and $y=u-v^2$, express $\frac{\partial z}{\partial v}$ in terms of $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, u, and v.

5. HORT For the surface defined by $x^5+y^5+z^3+xyz=0$, find an equation of the tangent plane at $(1,1,-1)$, and the value of $\frac{\partial z}{\partial x}$ at $(1,1,-1)$.

6. HUANG For the path defined by $\mathbf{f}(t)=\sin 2t\mathbf{i}+t\mathbf{j}+t^3\mathbf{k}$,
a) Set up (do not evaluate) an integral for the length of the path from $(0,0,0)$ to $(0,\pi,\pi^3)$.
b) Find the angle between $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and the path at $(0,0,0)$.

7. JACKSON Let $\mathbf{A}=<1,3,-2>$ and $\mathbf{B}=<-1,2,1>$. Find
a) a unit vector in the same direction as $\mathbf{A}+2\mathbf{B}$.
b) vectors $\mathbf{A}_1$ and $\mathbf{A}_2$ so that $\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2$, $\mathbf{A}_1\|\mathbf{B}$, and $\mathbf{A}_2\perp\mathbf{B}$.
c) the area of the parallelogram in $\mathbb{R}^3$ formed by $\mathbf{A}$ and $\mathbf{B}$.

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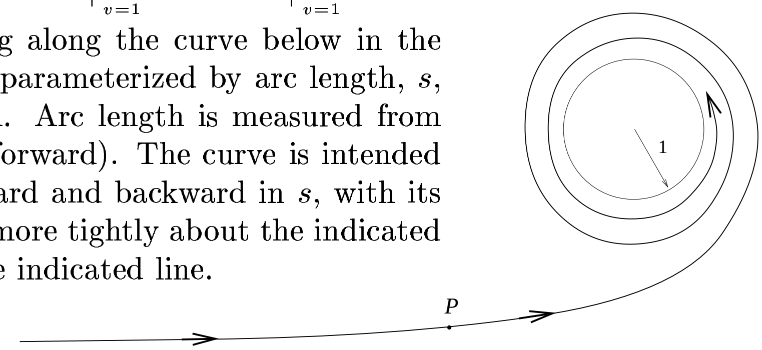
8. **KRAVTSOV** Use linear approximation at the point $(64, 9)$ for an appropriate function of two variables to approximate $\sqrt[3]{63}\sqrt{10}$.

9. **LUNEMANN** Find an equation of the tangent plane to the surface $z = x^3y$ at $(1, 2, 2)$.

10. **ODOI** Suppose that $w = f(x, y)$, where f is a function satisfying $f(1, 2) = 3$, $f_x(1, 2) = 1$, $f_y(1, 2) = -2$, $f_{xx}(1, 2) = 3$, $f_{xy}(1, 2) = 2$, and $f_{yy}(1, 2) = 0$. Suppose further that $x = u + v - 1$ and $y = 3uv - 1$. Find $\left.\frac{\partial w}{\partial u}\right|_{\substack{u=1\\v=1}}$ and $\left.\frac{\partial^2 w}{\partial v \partial u}\right|_{\substack{u=1\\v=1}}$.

11. **PERGAMENT** A point is moving along the curve below in the direction indicated. Its motion is parameterized by arc length, s, so that it is moving at unit speed. Arc length is measured from the point P (both backward and forward). The curve is intended to continue indefinitely both forward and backward in s, with its forward motion curling more and more tightly about the indicated circle, and, backward, closer to the indicated line.



Sketch a graph of the curvature, κ, as a function of the arc length, s. What are $\lim_{s\to+\infty} \kappa(s)$ and $\lim_{s\to-\infty} \kappa(s)$? Use complete English sentences to briefly explain the numbers you give.

12. **REINECKE** Find the unit tangent vector $\mathbf{T}$, the principal unit normal vector $\mathbf{N}$, and the curvature κ for the plane curve $\mathbf{r}(t) = (\cos(2t) + 2t\sin(2t))\,\mathbf{i} + (\sin(2t) - 2t\cos(2t))\,\mathbf{j}$ when $t > 0$.

13. **RYSLIK** Find & classify as well as you can all critical points of $f(x, y) = x^2 - 2yx^2 + 2y^2$.

14. **SULLIVAN** Find & classify as well as you can all critical points of $f(x, y) = (x^2 + y^2 - 1)^{10}$.

15. **TAKHTOVICH** Suppose the curve C has position vector $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t^2\mathbf{k}$.
a) What is the unit tangent vector to C when $t = \pi$?
b) Write an integral for the length of the part of C between $t = 0$ and $t = 2\pi$. DO NOT EVALUATE THE INTEGRAL.
c) The curvature of this curve is given by the formula $\kappa(t) = \sqrt{\frac{5+4t^2}{(4t^2+1)^3}}$ which you should *not* verify. Assume it is correct. Even though the first two components of this curve describe uniform circular motion, $\lim_{t\to\infty} \kappa(t) = 0$. Explain briefly why this can happen, using complete English sentences possibly assisted by properly labelled diagrams.

16. **TOKAYER** Use the ϵ-δ definition to verify that $g(x, y) = x^2y^2$ is continuous at $(-1, 2)$.

17. **VANDER VALK** Find the following limits, if they exist. Justify your answers!
a) $\lim_{(x,y)\to(0,0)} \frac{10x^4y}{x^4+y^4}$ b) $\lim_{(x,y)\to(0,0)} \frac{10x^3y}{x^4+y^4}$

18. **WALSH** Suppose that $G(u, v)$ is a differentiable function of two variables and that $g(x, y) = G\left(\frac{x}{y}, \frac{y}{x}\right)$. Show that $xg_x(x, y) + yg_y(x, y) = 0$.

19. **ZHU** a) Find parametric equations for the line through the points $(3, 2, 7)$ and $(-1, 1, 2)$.
b) At what point does this line intersect the plane $x + y + z = 22$?