Useful information concerning the FINAL EXAM in Math 291, Fall 2002

The place, day, and time of the exam will be:

Hill Center 525, Monday, December 23, 12-3 PM

The final exam will be cumulative, covering all of the material in the course. There will be slightly more emphasis on material discussed since the last exam. The exam rules will be unchanged. I do want to go home before sundown, though!

I'll have a **review session** at 4 PM on Sunday, December 22, in Hill 525.

Below are problems from past Math 291 exams which ask about material covered since the last exam. **Answers** to these problems should soon be available on the course web page.

1. Compute the following integrals. Use Green's theorem, Stokes' theorem or the divergence theorem wherever they are helpful.

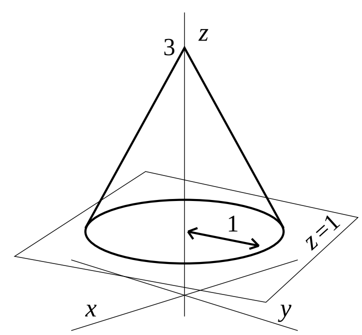
a) $\iint_D xy\,dA$, where D is the triangle in the xy-plane with vertices (0,0), (2,0), and (0,2).

b) $\int_C \mathbf{F}\cdot d\mathbf{s}$, where $\mathbf{F}(x,y) = x^2\mathbf{i} - xy\mathbf{j}$ and C is the segment of the parabola $y = x^2$ beginning at $(-1,1)$ and ending at $(1,1)$.

c) $\iint_S (x^3\mathbf{i} + y^3\mathbf{j} + \cos xy\mathbf{k})\cdot\mathbf{n}\,dS$, where S is the unit sphere and $\mathbf{n}$ points inward.

d) $\iint_S z^2\,dS$, where S is the surface $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$.

e) $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz^2\mathbf{j} + z^3\mathbf{k}$ and S is the lateral surface of the cone as shown, with $\mathbf{n}$ pointing outward.



Note I didn't get the picture from the instructor who gave me this problem, so I drew what I hope is a suitable picture here. I think the problem can be done with a standard result of vector calculus (and maybe even by a heroic direct computation!).

2. Compute $\int_C e^x \sin z\,dx + y^2\,dy + e^x\cos z\,dz$, where C is the oriented curve $\mathbf{x}(t) = (\cos t)^3\,\mathbf{i} + (\sin t)^3\,\mathbf{j} + t\mathbf{k}$, $0 \le t \le \pi/2$. First find a potential function.

3. A fluid has density 1500 and velocity field $\mathbf{v} = -y\,\mathbf{i} + x\,\mathbf{j} + 2z\,\mathbf{k}$. Find the flow outward through the sphere $x^2+y^2+z^2 = 25$.

4. Sketch the region E contained between the surfaces $z = x^2+y^2$ and $z = 2 - x^2 - y^2$ and let S be the boundary of E.

a) Find the volume of E.

b) Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{n}$ is the outer normal to S.

Please also look at the previous exams and review material in our course.
If I thought it was important then, I probably still think it is important!

You additionally may find useful a number of examinations on the subject matter of our course linked to the following web page (answers are separately supplied there):

`http://www-math.mit.edu/~jsantos/1802/exams.html`