Please write solutions to two of these problems. Hand them in Wednesday, September 25. The written solutions should be accompanied by explanations using complete English sentences. You may select at most **one** *of the first two problems. Students may work alone or in groups. All students working in a group should contribute to the writeup and should sign what is handed in.*

1. Find equations for two orthogonal planes both of which contain the line $\mathbf{v} = (1,0,3) + t(-1,2,1)$, one of which passes through the origin.

2. Find the distance between the pair of skew lines given below ("skew" in this case means a bit more than non-intersecting – it means that they are non-intersecting *and* not parallel):

$$\text{The line } L_1 \text{ is } \begin{cases} x = 2t+1 \\ y = -t-1 \\ z = 3t \end{cases}. \quad \text{The line } L_2 \text{ is } \begin{cases} x = 3s+2 \\ y = 5s-2 \\ z = -4 \end{cases}.$$

Comments: I used different letters (t and s) for the parameters in the two lines. This was to help, because these letters are "dummy" variables (similar logically to the letters in a definite integral: surely $\int_0^1 w^2 dw$ and $\int_0^1 u^2 du$ are the same).

3. Is the point (1,2,3) on a tangent line of the twisted cubic $\mathbf{c}(t) = (t, t^2, t^3)$?

4. A function $c : \mathbb{R} \to \mathbb{R}^2$ is called *smooth* if c is differentiable. Physically such functions should represent motion that has no jerks or kinks.

a) Suppose $q : \mathbb{R} \to \mathbb{R}$ is defined by $q(x) = \begin{cases} x^{100} & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$. Sketch $y = q(x)$ and prove that q is differentiable. (Do this by first computing the derivative algorithmically for $x < 0$ and $x > 0$ – this is easy. Compute $q'(0)$ by looking directly at the definition and computing the limit from both sides.)

b) Sketch the smooth curve defined by $c(t) = (q(t), q(-t))$.

c) Explain why you've drawn a picture of smooth motion. (Hardest part of the question!)

5. a) Compute the curvature of the plane curve $\begin{cases} x(t) = \dfrac{1-t^2}{1+t^2} \\ y(t) = \dfrac{2t}{1+t^2} \end{cases}$. Explain the result using reasoning independent of the curvature computation.

b) Compute the curvature of the space curve $\begin{cases} x(t) = \dfrac{2-t^2}{1+t^2} \\ y(t) = \dfrac{2t^2-2}{1+t^2} \\ z(t) = \dfrac{3t^2-2}{1+t^2} \end{cases}$. Explain the result using reasoning independent of the curvature computation.