#5 **Problems for 291:01** Fun with the chain rule. 10/6/2002

Please write solutions to two of these problems. Hand them in Monday, October 14. The written solutions should be accompanied by explanations using complete English sentences. Students should work alone. They may ask me questions.

1. a) Suppose $h(x,y) = x^y$. Find the domain of h and the first partial derivatives of h. I know that $2^3 = 8$. If x is increased by .01 (so (x,y) changes from $(2,3)$ to $(2.01,3)$), approximate the change in h using the linear approximation. If y is increased by .01 (so (x,y) changes from $(2,3)$ to $(2,3.01)$), approximate the change in h using the linear approximation. Compare the "exact answers" to the linearization answers.

b) Suppose $j(x,y,z) = x^{(y^z)}$. Find the domain of j and the first partial derivatives of j. I know that $2^{(3^4)} \approx 2.417 \cdot 10^{24}$* If one of the variables is increased by .01, which variable will likely make the biggest change in the value of j? Support your assertion by an argument using linear approximation based on the derivatives which have been calculated.

2. a) Suppose $f(u,v)$ is a differentiable function of u and v, and that $u = e^x \cos y$ and $v = e^x \sin y$. Express $\frac{\partial f}{\partial x}$ in terms of $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial v}$, and functions involving x and y.

b) Note that if $x = 0$ and $y = \frac{\pi}{2}$ then $u = 0$ and $v = 1$. Suppose also that you know $\frac{\partial f}{\partial u}(0,1) = 7$ and $\frac{\partial f}{\partial v}(0,1) = -3$. Use this information together with your formula in a) to compute $\frac{\partial f}{\partial x}$ when $x = 0$ and $y = \frac{\pi}{2}$.

3. Suppose $R(x,y) = v(x+y^2)$ where v is a four times differentiable function of *one* variable. Suppose you know also that:

$$v(0) = \alpha,\ v'(0) = \beta,\ v^{(2)}(0) = \gamma,\ v^{(3)}(0) = \delta,\ v^{(4)}(0) = \epsilon$$

Compute the seven quantities:

$$R(0,0),\ \frac{\partial R}{\partial x}(0,0),\ \frac{\partial R}{\partial y}(0,0),\ \frac{\partial^2 R}{\partial x^2}(0,0),\ \frac{\partial^2 R}{\partial y^2}(0,0),\ \frac{\partial^2 R}{\partial x \partial y}(0,0),\ \frac{\partial^4 R}{\partial x^2 \partial y^2}(0,0)$$

in terms of α, β, γ, δ, and ϵ.

4. Suppose $g(t) = Q(t^3, t^5)$. Suppose you also know that

$$Q(1,1) = A,\ \frac{\partial Q}{\partial x}(1,1) = B,\ \frac{\partial Q}{\partial y}(1,1) = C,\ \frac{\partial^2 Q}{\partial x^2}(1,1) = D,\ \frac{\partial^2 Q}{\partial x \partial y}(1,1) = E,\ \frac{\partial^2 Q}{\partial y^2}(1,1) = F$$

Compute the quantities $g(1)$, $g'(1)$, and $g''(1)$ in terms of A, B, C, D, and E.

5. If $\mathcal{M}$ is the set of 2×2 matrices ($\mathcal{M}$ is just a copy of $\mathbb{R}^4$), and $\det\colon \mathcal{M} \to \mathbb{R}$ is defined by $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$ then what is $\nabla \det$? Suppose A is the matrix $\begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}$ which has determinant -22. If we change the entries of the matrix by a little bit, where "little bit" means changes in the entries so that $(\triangle a)^2 + (\triangle b)^2 + (\triangle c)^2 + (\triangle a)^2 \le .01$, what changes are likely to *increase* the determinant of the resulting matrix as much as possible?

* Actually, it is *exactly* 24178 51639 22925 83494 12352.