#7 **Problems for 291:01** 10/28/2002

Please write solutions to two of these problems. Hand them in Monday, November 4. The written solutions should be accompanied by explanations using complete English sentences. Students may work in groups of at most two. They may ask me questions.

Multiple integrals without applications can initially seem rather empty. What is really new about computing $\int_1^2 \int_x^{x^2} 3x^2y - 2y^3\,dx\,dy$? It is just an iterated integral – involving repetition of operations done in one variable calculus. There is some new trickery, though, and some new problems with grammar, and improper integrals occur very often. And sometimes the definition is not so easy to understand.

1. **Impropriety** Suppose $\mathcal{S}$ is the unit square where $0 \le x \le 1$ and $0 \le y \le 1$. Does

$$\int\!\!\int_{\mathcal{S}} \frac{1}{x+y}\,dA$$

converge, and, if it does, what is its value? Include a simple sketch produced by `Maple` of the volume indicated in this integral.

2. **Grammar** Some of these iterated integrals correspond to real geometric problems (computation of volumes) and some do not. Some are actually illegal! Please indicate which are good and which are not. Explain your answers. Compute any iterated integral which corresponds to a volume. Include a sketch produced by `Maple` of each volume computed.

a) $\displaystyle\int_0^{x^3} \int_0^{y} x^4 + y^2 + 7\,dx\,dy$ b) $\displaystyle\int_0^{1} \int_0^{5x} x^4 + y^2 + 7\,dx\,dy$

c) $\displaystyle\int_5^{7} \int_{y^3}^{3y} x^4 + y^2 + 7\,dx\,dy$ d) $\displaystyle\int_{-1}^{0} \int_{-y^2}^{2y} x^4 + y^2 + 7\,dx\,dy$

3. **Trickery** Compute:

a) $\displaystyle\int_0^{\frac{\pi}{2}} \left(\int_y^{\frac{\pi}{2}} \frac{\sin x}{x}\,dx \right) dy$ b) $\displaystyle\int_0^{1} \left(\int_{\sqrt{y}}^{1} e^{(7x^3)}\,dx \right) dy$ c) $\displaystyle\int_0^{1} \left(\int_x^{x^{1/3}} \sqrt{1-y^4}\,dy \right) dx$

Hint: write 'em as a double integral, then re-iterate*.

4. **The Definition** Suppose $\mathcal{C}$ is the exponential spiral curve in $\mathbb{R}^2$ given by $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$ and suppose that F is the function defined by $F(x,y) = \begin{cases} 1 & \text{if } (x,y) \text{ is on } \mathcal{C} \\ 0 & \text{if } (x,y) \text{ is } \textit{not} \text{ on } \mathcal{C} \end{cases}$.

Does $\displaystyle\int_{-1}^{1} \int_{-1}^{1} F(x,y)\,dx\,dy$ exist? Why or why not? (Give some discussion supporting your assertion – discussion that I'll believe.) If it does, what is its value?

5. **Impropriety** Does either of these converge? If not, explain why. If yes, evaluate. Of course, explain your answers.

a) $\displaystyle\int\!\!\int_{\mathbb{R}^2} e^{-|x|-|y|}\,dA$ b) $\displaystyle\int\!\!\int_{\mathbb{R}^2} e^{-|x+y|}\,dA$

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* Yes, this is supposed to be a somewhat incomprehensible clue.