#8 **Problems for 291:01** 11/6/2002

Please write solutions to two of these problems. Hand them in Wednesday, November 13. The written solutions should be accompanied by explanations using complete English sentences. Students may work in groups of at most two. They may ask me questions.

1. Consider, as we did in class, the tetrahedrom T whose corners are $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$. Suppose the function f is defined by $f(x,y,z) = x^3 + y^2 + z$. The goal of this problem is to use `Maple` to compute $\iiint_T f\,dV$

a) Write the integral as an iterated integral in the order $dx\,dy\,dz$. Have `Maple` do each of the integrals, and present a printout of the computations.

b) Write the integral as an iterated integral in the order $dy\,dz\,dx$. Have `Maple` do each of the integrals, and present a printout of the computations.

c) Write the integral as an iterated integral in the order $dz\,dx\,dy$. Have `Maple` do each of the integrals, and present a printout of the computations.

d) $\Big($Optional$\Big)$ Write an essay of at most 100 words describing the irritation you'd have computing this yourself.

2. Compute the triple integral of $x^3 + y^2 + z$ over the **smaller** volume enclosed by the plane $z = 3$, the plane $z = 3x + \frac{3}{2}y$, and the paraboloid $z = 3x^2 + \frac{3}{4}y^2$. You may have `Maple` do the computations, but you must describe how to set up the integrals, and also give a printout of the computations.

3^-. **Warmup for problem 3*** For which γ does

$$\int_0^1 t^\gamma\,dt$$

converge, and, for those γ, what is the value of the integral?

3. **Impropriety again** Suppose $\mathcal{A}$ is the region inside the unit square bounded by the curves $y = x$ and $y = x^2$. Suppose $\mathcal{B}$ is the region inside the unit square bounded by the curves $y = x^2$ and $y = 0$ and $x = 1$. (Simple pictures might help you here.)

a) For which α and β does

$$\iint_{\mathcal{A}} x^\alpha y^\beta\,dA$$

converge, and, for those α and β, what is the value of the integral?

b) For which α and β does

$$\iint_{\mathcal{B}} x^\alpha y^\beta\,dA$$

converge, and, for those α and β, what is the value of the integral?

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* You don't need to hand this in but you should "recall" the answer from previous calculus experience.

4. I want to set up a triple integral to compute the mass of a solid in $\mathbb{R}^3$. The density of the solid at the point (x, y, z) is $\rho(x, y, z)$ (a function which is positive at every point) and the mass will be given by some formula like this:

$$\text{Mass } = \int_?^? \int_?^? \int_?^? \rho(x, y, z)\, d?\, d?\, d?$$

You must choose an order of integration <u>and</u> the three integral signs with limits. The integral signs should give a standard description of a volume in $\mathbb{R}^3$, and they should be selected from among the following:

$$\int_x^{x^3} \qquad \int_{x^2y^4}^{5xy^2} \qquad \int_{-1}^{-2} \qquad \int_{\frac{1}{x^2y^4}}^{\frac{1}{5xy^2}} \qquad \int_0^{\sin(100x)} \qquad \int_{-5xy}^{-(x+y)} \qquad \int_1^2$$

Write the order you choose and the limits on the integrals you choose. You can't compute the mass since I haven't told you ρ. Just set up the triple integral, and *explain why* your choice is valid. (I think there's only one correct way to answer this question.)

5. a) Verify the equation (that is, do the computation)

$$\int_0^1 \int_0^{1-x} \exp\left(\frac{y}{x+y}\right)\, dy\, dx = \frac{e-1}{2}$$

by using the transformation $\begin{cases} x + y = u \\ y = uv \end{cases}$.*

b) Use a linear change of variables to find the area inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (Try $au = x$ and $bv = y$. Please show how the change of variables formula works in this case.)

* This problem is from the Schaum's outline series problem book on advanced calculus.