(12) 1. Suppose that the position of a point in $\mathbb{R}^2$ is given by $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$.

a) Carefully compute the velocity vector, $\mathbf{v}(t)$, and the acceleration vector, $\mathbf{a}(t)$.
b) Compute the length of the curve from $t = 0$ to $t = 2\pi$.
Comment Your answer(s) should be exact. Answers may use traditional mathematical constants such as π and e and operations involving arithmetic and root extraction.c) Compute the angle between the position vector and the acceleration vector, and show that the angle does not depend upon t. What is the angle?

(10) 2. a) If $F(x,y) = \dfrac{3x - 4y}{\sqrt{x^2 + y^2}}$, briefly explain why $\displaystyle\lim_{(x,y)\to(0,0)} F(x,y)$ does not exist.
b) If $G(x,y) = \dfrac{3x^2 - 4y^2}{\sqrt{x^2 + y^2}}$, briefly explain why $\displaystyle\lim_{(x,y)\to(0,0)} G(x,y)$ exists.

(10) 3. Suppose that f is a differentiable function of *one* variable. If $z = f\left(\dfrac{xy}{x^2 + y^2}\right)$ prove that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$.

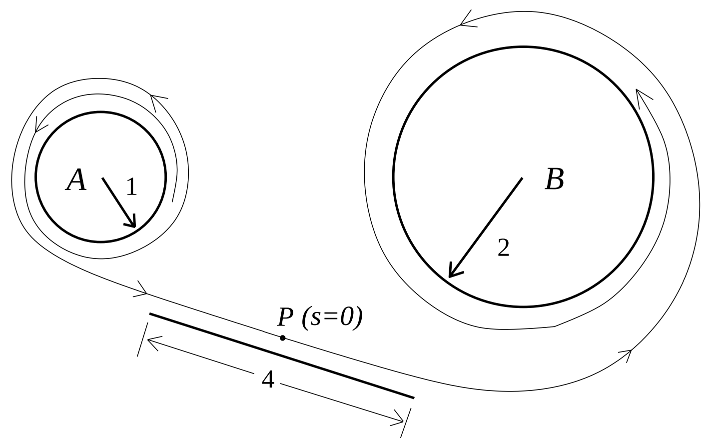
(12) 4. Find all critical points of the function $K(x,y) = (y^2 + x)e^{(-x^2/2)}$. Describe (as well as you can) the type of each critical point. Explain your conclusions.

(12) 5. Suppose $f(x,y,z) = x^3 + y^2 z$.
a) Find an equation of the tangent plane for the level surface of f which passes through $(2, 1, -3)$.
You do **not** need to "simplify" your answer!
b) In what direction will f increase most rapidly at $(2, 1, -3)$? Write a unit vector in that direction.
You do **not** need to "simplify" your answer!c) What is the directional derivative of f at $(2, 1, -3)$ in the direction found in b)?
You do **not** need to "simplify" your answer!

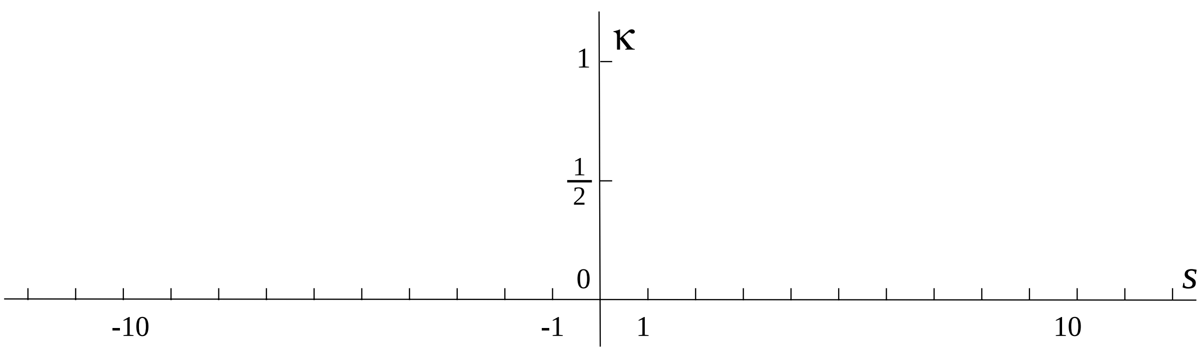
(12) 6. Suppose $f(x,y) = \begin{cases} x & \text{if } x > 0 \\ 2y & \text{if } x \le 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$.

a) For which (x,y) in $\mathbb{R}^2$ is f **not** continuous?
(Just write your answer carefully. You need *not* give supporting reasons.)
b) There is $H > 0$ so that if $\|(x,y) - (0,0)\| < H$ then $|f(x,y) - f(0,0)| < \dfrac{1}{1{,}000}$.
Find such an $H > 0$ and explain *why* your assertion is correct.
Note Any correct $H > 0$ is acceptable, but verification must be given.

(12) 7. A point is moving along the curve below in the direction indicated. Its motion is parameterized by arc length, s, so that it is moving at unit speed. Arc length is measured from the point P (both backward and forward). The curve is intended to continue indefinitely both forward and backward in s, with its forward motion curling more and more tightly around the indicated circle, B, and, backward, curling more and more tightly around the other circle, A. Near P the curve is parallel to the indicated line segment.



Sketch a graph of the curvature, κ, as a function of the arc length, s. What are $\lim_{s\to+\infty} \kappa(s)$ and $\lim_{s\to-\infty} \kappa(s)$? Use complete English sentences to briefly explain the numbers you give.



Note that the units on the horizontal and vertical axes differ in length.

$\lim_{s\to-\infty} \kappa(s) =$ ______ $\lim_{s\to+\infty} \kappa(s) =$ ______

(12) 8. The polynomial equation $f(x,y,z) = 2xy + x^2 + 5y^3z + z^4 = 6$ is satisfied by the point $p = (0,-1,2)$ (be careful of the order of the variables – check that this is correct by substituting!). Suppose now that we change the first two coordinates of p and get a point $q = (.03, -1.05, \ldots)$. Use linear approximation to find an approximate value for the third (z) coordinate of q if q also satisfies the equation $f(x,y,z) = 6$.
You do **not** need to "simplify" your answer!

(8) 9. The vector $\mathbf{v}$ is $3\mathbf{i} - 7\mathbf{j} + \mathbf{k}$ and the vector $\mathbf{w}$ is $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$. Write $\mathbf{v}$ as a sum of two vectors, $\mathbf{v}_{\|}$ and $\mathbf{v}_{\perp}$, where $\mathbf{v}_{\|}$ is a scalar multiple of $\mathbf{w}$ and $\mathbf{v}_{\perp}$ is a vector orthogonal to $\mathbf{w}$.

First Exam for Math 291, section 1

March 13, 2003

NAME ____

Do all problems, in any order.
No notes or texts may be used on this exam.
You may use a calculator during the last 20 minutes of the exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	10	
3	10	
4	12	
5	12	
6	12	
7	12	
8	12	
9	8	
Total Points Earned:		